Maximum Transmission Constraint Equation for Note 229 (5)

This note generates a constraint equation for the problem set up in note 229(5) for the specific case of

$\frac{dT}{d\theta} = 0$

which occurs when

$\theta = \frac{1}{2}$

This solves the equation developed for this constraint developed in the previous calculation,

$f(u) = \text{Ln}\{2)$

```
f1 = 1;
f2 = λ / 2;
f3 = -β u / 2;
f4 = (1 / 2) / (1 + α Exp[u]);
f = f1 + f2 + f3 + f4
```

$$\frac{\lambda}{2} + \frac{1}{2(\alpha e^u + 1)} - \frac{\beta u}{2} + 1$$

```
c1 = Integrate[f1, {u, 1, γ}]
c2 = Integrate[f2, {u, 1, γ}]
c3 = Integrate[f3, {u, 1, γ}]
```

$$\gamma - 1$$

$$\frac{1}{2}(\gamma - 1)\lambda$$

$$\frac{\beta}{4} - \frac{\beta\gamma^2}{4}$$

```
c4 = Integrate[f4, {u, 1, γ}]
```

$$\frac{1}{2}(-\log(\alpha e^\gamma + 1) + \log(e\,\alpha + 1) + \gamma - 1)$$

```
total = Simplify[c1 + c2 + c3 + c4]
```

$$\frac{1}{4}(-2\log(\alpha e^\gamma + 1) + 2\log(e\,\alpha + 1) - (\gamma - 1)(\beta\gamma + \beta - 2\lambda - 6))$$

```
total /. γ → b / a;
% /. α → Exp[-R0 / a];
% /. β → z1 z2 q^2 / (4 Pi ϵ v0);
% /. λ → e0 / v0
```

$$\frac{1}{4}\left(-\left(\frac{b}{a} - 1\right)\left(\frac{b\,q^2\,\text{z1 z2}}{4\pi a\,\text{v0}\,\epsilon} - \frac{2\,\text{e0}}{\text{v0}} + \frac{q^2\,\text{z1 z2}}{4\pi\,\text{v0}\,\epsilon} - 6\right) - 2\log\left(e^{\frac{b}{a} - \frac{\text{R0}}{a}} + 1\right) + 2\log\left(e^{1 - \frac{\text{R0}}{a}} + 1\right)\right)$$

```
lhs = FullSimplify[%]
```

$$\frac{8\pi a^2\,\text{v0}\,\epsilon\left(\log\left(e^{1-\frac{\text{R0}}{a}} + 1\right) - \log\left(e^{\frac{b-\text{R0}}{a}} + 1\right)\right) + (a - b)(q^2\,\text{z1 z2}\,(a + b) - 8\pi a\,\epsilon\,(\text{e0} + 3\,\text{v0}))}{16\pi a^2\,\text{v0}\,\epsilon}$$

```
k1 = Sqrt[2 μ v0] / (ℏ a);
rhs = FullSimplify[Log[2] / k1]
```

$$\frac{a\,\hbar\,\log(2)}{\sqrt{2}\,\sqrt{\mu\,\text{v0}}}$$

```
eqn = FullSimplify[lhs - rhs]
```

$$\frac{8\,\pi\,a^2\,\text{v0}\,\epsilon\left(\log\left(e^{1-\frac{\text{R0}}{a}}+1\right)-\log\left(e^{\frac{b-\text{R0}}{a}}+1\right)\right)+(a-b)\left(q^2\,\text{z1}\,\text{z2}\,(a+b)-8\,\pi\,a\,\epsilon\,(\text{e0}+3\,\text{v0})\right)}{16\,\pi\,a^2\,\text{v0}\,\epsilon}-\frac{a\,\hbar\,\log(2)}{\sqrt{2}\,\sqrt{\mu\,\text{v0}}}$$