1)

Note 242(5): Evaluation of the True Anomaly for a Given Distance between m and M

The true anomaly in general is given by:

$$\theta = \frac{L_0}{\sqrt{2}\, m r^2} \int \left(- \int r \Omega^2 dr - A \right)^{-1/2} dr + B \quad -(1)$$

In order to evaluate θ for a distance:

$$r = R \quad -(2)$$

the correct formula is:

$$\theta = \frac{L_0}{\sqrt{2}\, m R^2} \int_0^R \left(- \int r \Omega^2 dr - A \right)^{-1/2} dr + B \quad -(2)$$

where

$$\Omega^2 = -\frac{L_0^2}{m^2 r^4} - \frac{F(r)}{m r} \quad -(3)$$

The inner integral in eq. (2) must be an indefinite integral, so the resulting integration is a function of r. Therefore:

$$\theta = \frac{L_0}{\sqrt{2}\, m R^2} \int_0^R f(r)\, dr + B \quad -(4)$$

$$f(r) = \left(- \int r \Omega^2 dr - A \right)^{-1/2} \quad -(5)$$

2) In general $\theta(r)$ is given as a function of r by eq. (1) for any force $F(r)$.

In the Newtonian theory:

$$F(r) = -\frac{mMG}{r^2} \quad -(6)$$

and

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(7)$$

so

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(8)$$

and

$$\theta(r) = \cos^{-1}\left(\frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)\right) \quad -(9).$$

This gives another baseline check of the method because eqs. (1) and (9) must be the same if $F(r)$ is given by eq. (6) and if:

$$L_0^2 = \alpha m^2 M G. \quad -(10)$$