1 Calculation of x

```
(%i1) kill(all);
```
(%o0) *done*

```
(%i1) F1: -gamma^4*m*M*G/r^2 - gamma^2*alpha*m*M*G/r^3*(1-gamma^2);
```
(%o1) $$-\frac{m\,G\,M\,\Gamma^4}{r^2}-\frac{\alpha\,m\,G\,M\,\Gamma^2(1-\Gamma^2)}{r^3}$$

```
(%i2) F2: -m*M*G*x^2/r^2-alpha*(1-x^2)*m*M*G/r^3;
```
(%o2) $$-\frac{m\,x^2\,G\,M}{r^2}-\frac{\alpha\,m(1-x^2)G\,M}{r^3}$$

```
(%i3) x2: solve(F1=F2, x^2);
```
(%o3) $$[x^2=\frac{(r-\alpha)\Gamma^4+\alpha\,\Gamma^2-\alpha}{r-\alpha}]$$

```
(%i4) x2: rhs(first(x2));
```
(%o4) $$\frac{(r-\alpha)\Gamma^4+\alpha\,\Gamma^2-\alpha}{r-\alpha}$$

```
(%i5) limit(x2,r,alpha);
```
(%o5) *infinity*

```
(%i6) limit(x2,gamma,1);
```
(%o6) 1

2 Simplification of gamma

```
(%i7) v2: M*G*(2/r-1/a);
```
(%o7) $$\left(\frac{2}{r}-\frac{1}{a}\right)G\,M$$

```
(%i8) a:alpha/(epsilon^2-1);
```
(%o8) $$\frac{\alpha}{\varepsilon^2-1}$$

```
(%i9) v2: ev (v2);
```
(%o9) $$\left(\frac{2}{r}-\frac{\varepsilon^2-1}{\alpha}\right)G\,M$$

```
(%i10) gamma: 1/sqrt(1-v2);
```
(%o10) $$\frac{1}{\sqrt{1-\left(\frac{2}{r}-\frac{\varepsilon^2-1}{\alpha}\right)G\,M}}$$

```
(%i11) MG: rhs(first(solve(L0^2=alpha*m^2*MG,MG)));
```

(%o11) $$\frac{L0^2}{\alpha\, m^2}$$

```
(%i12) assume(alpha>0,m>0);
```

(%o12) $[\alpha>0, m>0]$

```
(%i13) gamma: ratsubst(MG,G*M,gamma);
```

(%o13) $$\frac{\alpha\, m}{\sqrt{\frac{r\,(\varepsilon^2\, L0^2 - L0^2 + \alpha^2\, m^2) - 2\,\alpha\, L0^2}{r}}}$$

```
(%i14) gamma: radcan(gamma);
```

(%o14) $$\frac{\alpha\, m\,\sqrt{r}}{\sqrt{((\varepsilon^2 - 1)\, r - 2\,\alpha)\, L0^2 + \alpha^2\, m^2\, r}}$$

```
(%i15) v2: ratsubst(MG,G*M,v2);
```

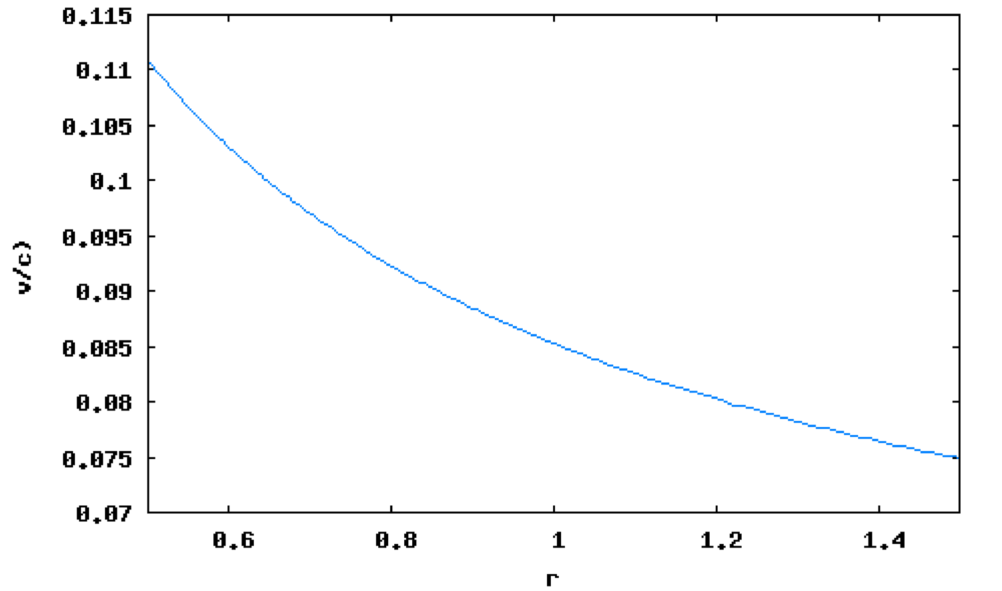
(%o15) $$-\frac{r\,(\varepsilon^2\, L0^2 - L0^2) - 2\,\alpha\, L0^2}{\alpha^2\, m^2\, r}$$

```
(%i16) v1: ev(radcan(sqrt(v2)), [alpha=1, m=1, epsilon=0.3, L0=0.05]);
rat: replaced -0.005 by -1/200 = -0.005
rat: replaced -0.002275 by -91/40000 = -0.002275
```

(%o16) $$\frac{\sqrt{91\, r + 200}}{200\,\sqrt{r}}$$

```
(%i17) wxplot2d([v1], [r,0.5,1.5],
[ylabel, "v/c)"])$
```

(%t17)

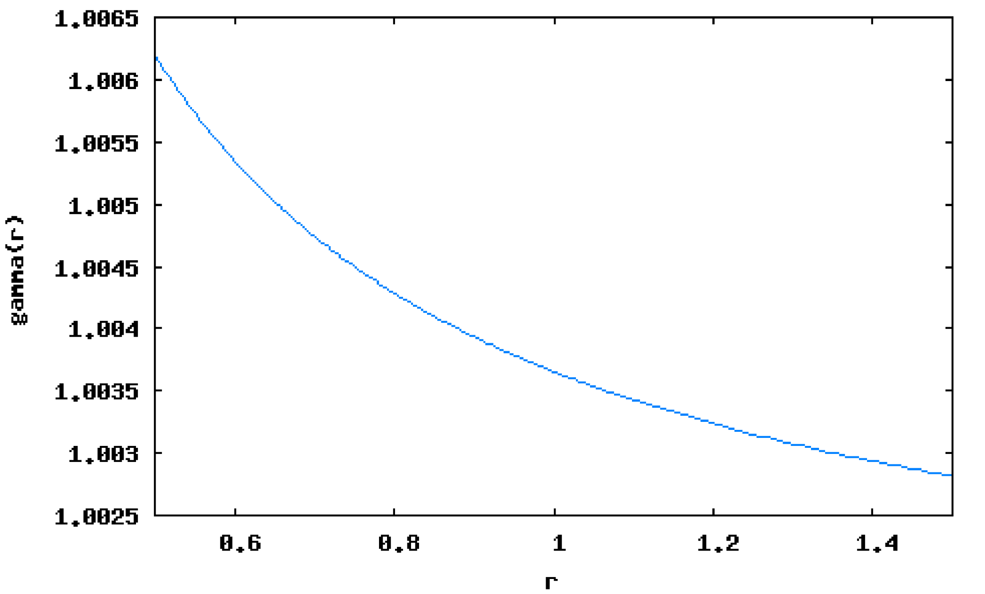


(%i18)
```
gamma1: ev(gamma, [alpha=1, m=1, epsilon=0.3, L0=0.05]);
```

(%o18) $$\frac{\sqrt{r}}{\sqrt{r+0.0025\left(-0.91\,r-2\right)}}$$

(%i19)
```
wxplot2d([gamma1], [r,0.5,1.5],
[ylabel, "gamma(r)"])$
```

(%t19)



(%i20)
```
gammax: ev(gamma,[r=alpha]);
```

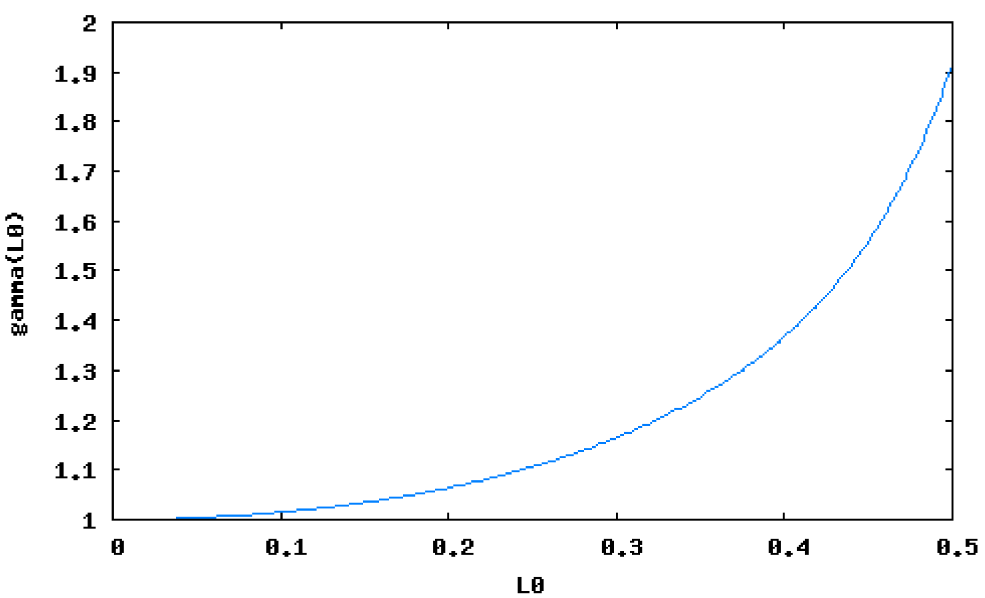
(%o20) $$\frac{\alpha^{3/2}\,m}{\sqrt{\left(\alpha\left(\varepsilon^2-1\right)-2\,\alpha\right)L0^2+\alpha^3\,m^2}}$$

(%i21)
```
gamma1: ev(gammax, [alpha=1, m=1, epsilon=0.3]);
```

(%o21) $$\frac{1}{\sqrt{1-2.91\,L0^2}}$$

(%i22)
```
wxplot2d([gamma1], [L0,0.0,0.5],
[ylabel, "gamma(L0)"])$
```

(%t22)



3 Plot of x(r)

(%i23) x2: (ev(x2));

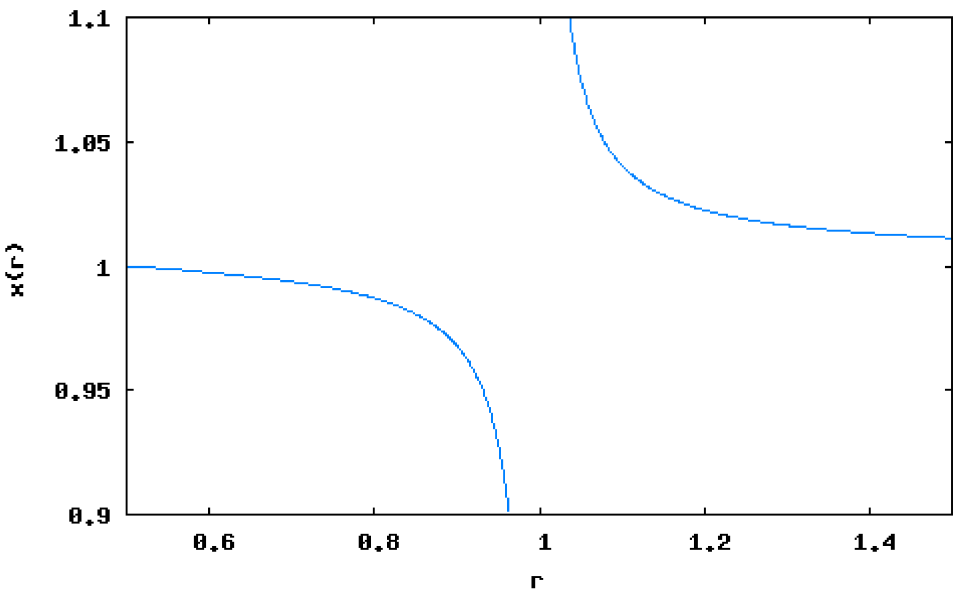
(%o23) $$\frac{\frac{\alpha^3 m^2 r}{((\varepsilon^2-1)r-2\alpha)L0^2+\alpha^2 m^2 r}+\frac{\alpha^4 m^4 r^2 (r-\alpha)}{(((\varepsilon^2-1)r-2\alpha)L0^2+\alpha^2 m^2 r)^2}-\alpha}{r-\alpha}$$

(%i24) x1: sqrt(ev(x2, [m=1, alpha=1, epsilon=0.3, L0=0.05]));

(%o24) $$\sqrt{\frac{\frac{r}{r+0.0025(-0.91r-2)}+\frac{(r-1)r^2}{(r+0.0025(-0.91r-2))^2}-1}{r-1}}$$

(%i25) wxplot2d([x1], [r,0.5,1.5],[y,0.9,1.1],
[ylabel, "x(r)"])$

plot2d: expression evaluates to non-numeric value somewhere in plotting rang
plot2d: some values were clipped.

(%t25) 

4 Integration of dr/dt

(%i26) assume(alpha>0);

(%o26) [*redundant*]

(%i27) r1: alpha/(1+epsilon*cos(x*theta));

(%o27) $$\frac{\alpha}{\varepsilon\cos(\theta x)+1}$$

(%i28) diff(r1,theta);

(%o28) $$\frac{\alpha\varepsilon x\sin(\theta x)}{(\varepsilon\cos(\theta x)+1)^2}$$

```
(%i29) drdth: ratsubst(r/alpha,r1/alpha,%);
```

(%o29) $$\frac{\varepsilon\, r^2\, x \sin(\theta\, x)}{\alpha}$$

```
(%i30) assume(epsilon>0, epsilon<1, r>0, m>0);
```

(%o30) $[\varepsilon>0,\varepsilon<1,r>0,redundant]$

```
(%i31) L0 = m*r^2*drdth^-1*drdt;
```

(%o31) $$L0=\frac{\alpha\, drdt\, m}{\varepsilon\, x \sin(\theta\, x)}$$

```
(%i32) solve(%, drdt);
```

(%o32) $$\left[drdt=\frac{\varepsilon\, x \sin(\theta\, x)\, L0}{\alpha\, m}\right]$$

```
(%i33) ratsubst(sqrt(1-1/epsilon^2*(alpha/r-1)^2), sin(theta*x), %);
```

(%o33) $$\left[drdt=\frac{\varepsilon\sqrt{1-\frac{\left(\frac{\alpha}{r}-1\right)^2}{\varepsilon^2}}\, x\, L0}{\alpha\, m}\right]$$

```
(%i34) ratsimp(%);
```

(%o34) $$\left[drdt=\frac{\sqrt{(\varepsilon^2-1)\, r^2+2\,\alpha\, r-\alpha^2}\, x\, L0}{\alpha\, m\, r}\right]$$

```
(%i35) dtdr: 1/rhs(first(%));
```

(%o35) $$\frac{\alpha\, m\, r}{\sqrt{(\varepsilon^2-1)\, r^2+2\,\alpha\, r-\alpha^2}\, x\, L0}$$

```
(%i36) integrate(dtdr,r);
```

(%o36) $$\frac{\alpha\, m\left(\frac{\alpha \operatorname{asin}\left(\frac{2\,(\varepsilon^2-1)\, r+2\,\alpha}{\sqrt{4\,\alpha^2\,(\varepsilon^2-1)+4\,\alpha^2}}\right)}{\sqrt{1-\varepsilon^2}\,(\varepsilon^2-1)}+\frac{\sqrt{(\varepsilon^2-1)\, r^2+2\,\alpha\, r-\alpha^2}}{\varepsilon^2-1}\right)}{x\, L0}$$

```
(%i37) radcan(%);
```

(%o37) $$\frac{\alpha^2\, m \operatorname{asin}\left(\frac{(\varepsilon^2-1)\, r+\alpha}{\alpha\,\varepsilon}\right)+\alpha\sqrt{1-\varepsilon}\,\sqrt{\varepsilon+1}\, m\sqrt{(\varepsilon-1)\, r+\alpha}\,\sqrt{(\varepsilon+1)\, r-\alpha}}{\sqrt{1-\varepsilon}\,\sqrt{\varepsilon+1}\,(\varepsilon^2-1)\, x\, L0}$$

Solution t(r)

(%i38) t : %;

(%o38) $$\frac{\alpha^2\, m\, \mathrm{asin}\left(\frac{(\varepsilon^2-1)\,r+\alpha}{\alpha\,\varepsilon}\right)+\alpha\sqrt{1-\varepsilon}\,\sqrt{\varepsilon+1}\,m\sqrt{(\varepsilon-1)\,r+\alpha}\,\sqrt{(\varepsilon+1)\,r-\alpha}}{\sqrt{1-\varepsilon}\,\sqrt{\varepsilon+1}\,(\varepsilon^2-1)\,x\,L0}$$

5 Plot t(r)

(%i39) t1: ev(t, [epsilon=0.3, x=1, L0=0.01, alpha=1, m=1]);

(%o39) $-115.1961359035075\,(0.95393920141695\sqrt{1-0.7\,r}\,\sqrt{1.3\,r-1}+\mathrm{asin}(3.333333333333334\,(1-0.91\,r)))$

(%i40) t2: ev(t, [epsilon=0.3, x=1, L0=0.03, alpha=1, m=1]);

(%o40) $-38.39871196783583\,(0.95393920141695\sqrt{1-0.7\,r}\,\sqrt{1.3\,r-1}+\mathrm{asin}(3.333333333333334\,(1-0.91\,r)))$

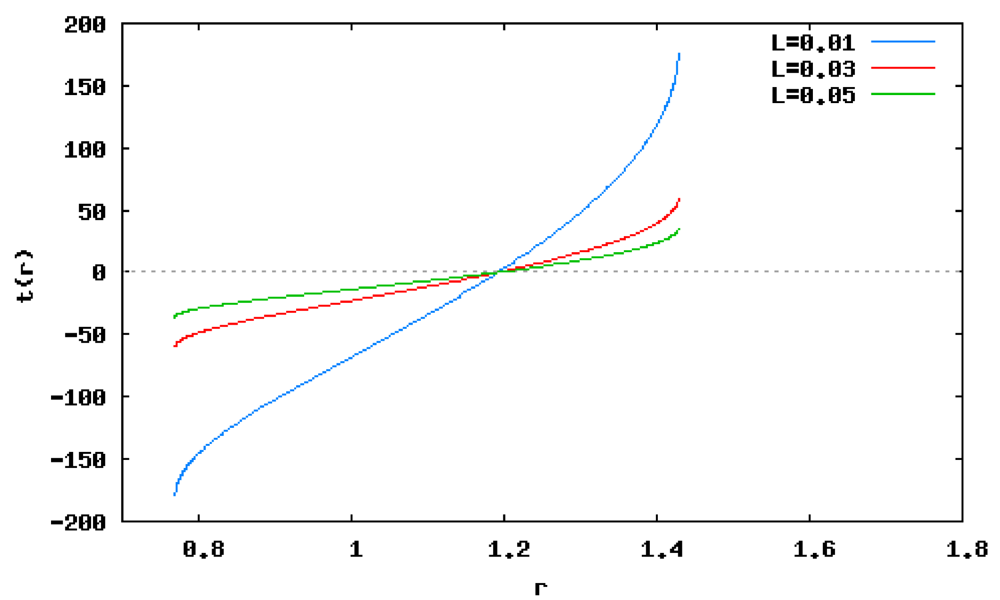
(%i41) t3: ev(t, [epsilon=0.3, x=1, L0=0.05, alpha=1, m=1]);

(%o41) $-23.0392271807015\,(0.95393920141695\sqrt{1-0.7\,r}\,\sqrt{1.3\,r-1}+\mathrm{asin}(3.333333333333334\,(1-0.91\,r)))$

(%i42) wxplot2d([t1,t2,t3], [r,0.7,1.8],
[ylabel, "t(r)"], [legend, "L=0.01", "L=0.03", "L=0.05"]);

plot2d: expression evaluates to non-numeric value somewhere in plotting rang
plot2d: expression evaluates to non-numeric value somewhere in plotting rang
plot2d: expression evaluates to non-numeric value somewhere in plotting rang

(%t42)



(%o42)