234(1): Computer Evaluation of the Orbit of Standard Einsteinian General Relativity.

The orbit is evaluated from the equation:

$$\left(\frac{du}{d\theta}\right)^2 = \frac{1}{b^2} - \left(1 - u r_s\right)\left(\frac{1}{a^2} + u^2\right) \quad -(1)$$

where

$$r_s = \frac{2mG}{c^2}. \quad -(2)$$

Eq. (1) is expressed as:

$$\left(\frac{du}{d\theta}\right)^2 = r_s\left(u - u_1\right)\left(u - u_2\right)\left(u - u_3\right), \quad -(3)$$

$$u_1 + u_2 + u_3 = \frac{1}{r_s}. \quad -(4)$$

The analytical solution is:

$$u = u_1 + \left(u_2 - u_1\right) sn^2\left(\frac{1}{2}\theta\left(r_s\left(u_3 - u_1\right)\right)^{1/2} + \delta\right) \quad -(5)$$

where sn is the sinus amplitudinus function, one of the Jacobi elliptic functions, and δ is a constant of integration that depends on the initial position. The elliptic modulus is:

$$k = \left(\frac{u_2 - u_1}{u_3 - u_1}\right)^{1/2} \quad -(6)$$

For an ellipse:

$$\frac{1}{u_1} = r_{max} = \alpha\left(1 + \epsilon\right) \quad -(7)$$

$$\frac{1}{u_2} = r_{min} = \alpha\left(1 - \epsilon\right) \quad -(8)$$

2)

So:

$$u = \frac{1}{r_{max}} + \left(\frac{1}{r_{min}} - \frac{1}{r_{max}}\right) sn^2\left(\frac{1}{2}\theta\left(r_s\left(u_3 - \frac{1}{r_{max}}\right)\right)^{1/2} + \delta\right) \quad -(9)$$

This can be evaluated directly with a program that can evaluate the Jacobi elliptical functions. The usual approximation assumes that:

$$u_3 \rightarrow \frac{1}{r_s} \gg u_1 \text{ or } u_2 \quad -(10)$$

but obviously, eq. (9) is not the true precessing ellipse

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(11)$$

in general.

In the standard method:

$$\Delta\theta = \frac{4K}{\left(r_s\left(u_3 - u_1\right)\right)^{1/2}} \quad -(12)$$

where

$$K = \int_0^1 \frac{dy}{\left(\left(1-y^2\right)\left(1-k^2y^2\right)\right)^{1/2}} \quad -(13)$$

$$k^2 = \frac{u_2 - u_1}{u_3 - u_1}. \quad -(14)$$

Therefore in general, $\underline{\Delta\theta}$ is a $\underline{\text{function}}$ of u_3.

3) For the true precessing ellipse:

$$\Delta\theta = 2\pi(x - 1) \quad -(15)$$

If $\Delta\theta$ from eq. (12) is plotted against $\Delta\theta$ from eq. (15) for any u_3 and x, the results will be completely different. This means that EGR does not give a precessing ellipse in general. Furthermore, $\Delta\theta$ from EGR is singular at

$$u_3 = u_1, \quad -(16)$$

and also at:

$$y^2 = 1 \quad -(17)$$

and

$$k^2 y^2 = 1. \quad -(18)$$

whereas the true $\Delta\theta$ from eq. (15) is never singular.

Finally eq. (15) gives the experimental precession very simply, in terms of x, and is the preferred theory by Ockham's Razor.

Reference . Wikipedia, "Schwarzschild geodesics"