233(3): Characteristics of the x Factor for the Minkowski Orbit

For this orbit:

$$x^2 = \left(\frac{d}{\epsilon}\right)^2 \frac{\left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2}\right)}{1 - \frac{1}{\epsilon^2}\left(\frac{d}{r} - 1\right)^2} \quad -(1)$$

where

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E} \quad -(2)$$

$$d = (1+\epsilon)\, r_{min} \quad -(3)$$

$$= (1-\epsilon)\, r_{max} \quad -(4)$$

Here, the Minkowski orbit is the precessing ellipse:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad -(5)$$

Under these conditions, special relativity produces the true precessing elliptical orbit.

In these equations E is the total energy, L is the total angular momentum, (r, θ) the plane polar coordinate system, c the speed of light in a vacuum, d the half right latitude, ϵ the ellipticity, r_{min} is the distance of closest approach of m to M and r_{max} the furthest distance apart. In eq. (1):

$$\frac{1}{b^2} - \frac{1}{a^2} = \frac{E^2}{c^2 L^2} - \frac{m^2 c^2}{L^2} \quad -(6)$$

$$= \left(E^2 - m^2 c^4\right)/L^2 = p^2/L^2$$

where p is the relativistic linear momentum.

2) In special relativity:

$$E = \gamma m c^2, \quad p = \gamma m v \quad -(7)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \quad -(8)$$

and the angular momentum is:

$$L = \gamma m r^2 \omega = \gamma m r^2 \frac{d\theta}{dt} \quad -(9)$$

where ω is the angular velocity.

Therefore

$$\left(\frac{p}{L}\right)^2 = \left(\frac{v}{r^2 \omega}\right)^2 \quad -(10)$$

and

$$x^2 = \left(\frac{d}{\epsilon}\right)^2 \frac{1}{r^2} \frac{\left(\left(\frac{v}{r\omega}\right)^2 - 1\right)}{1 - \frac{1}{\epsilon^2}\left(\frac{d}{r} - 1\right)^2} \quad -(11)$$

$$\boxed{x^2 = \frac{d^2\left(\left(\frac{v}{r\omega}\right)^2 - 1\right)}{\epsilon^2 r^2 - (d-r)^2}} \quad -(12)$$

the denominator can be factorized as follows:

$$\begin{aligned} \epsilon^2 r^2 - (d-r)^2 &= (\epsilon r + d - r)(\epsilon r - (d-r)) \\ &= (r(\epsilon - 1) + d)(r(\epsilon + 1) - d) \end{aligned} \quad -(13)$$

Using eqs. (3), (4) and (13):

3)

$$\epsilon^2 r^2 - (d-r)^2 = (1-\epsilon)(1+\epsilon)(r_{max} - r)(r - r_{min}) \quad -(14)$$

So:

$$x^2 = \frac{d^2\left(\left(\frac{v}{r\omega}\right)^2 - 1\right)}{(1-\epsilon)(1+\epsilon)(r_{max}-r)(r-r_{min})} \quad -(15)$$

where

$$d = (1+\epsilon)r_{min} = (1-\epsilon)r_{max} \quad -(16)$$

So:

$$\boxed{\begin{aligned} x^2 &= \left(\frac{1-\epsilon}{1+\epsilon}\right)\frac{r_{max}^2}{(r_{max}-r)(r-r_{min})}\left(\left(\frac{v}{r\omega}\right)^2 - 1\right) \\ &= \left(\frac{1+\epsilon}{1-\epsilon}\right)\frac{r_{min}^2}{(r_{max}-r)(r-r_{min})}\left(\left(\frac{v}{r\omega}\right)^2 - 1\right) \end{aligned}}$$

$$-(17)$$

In the Newtonian limit:

$$x^2 = 1 \quad -(18)$$

so eq. (17) gives:

4)

$$\left(\frac{v}{r\omega}\right)^2 - 1 = \left(\frac{1+\epsilon}{1-\epsilon}\right)\frac{(r_{max}-r)(r-r_{min})}{r_{max}^2}$$

$$= \left(\frac{1-\epsilon}{1+\epsilon}\right)\frac{(r_{max}-r)(r-r_{min})}{r_{min}^2} \qquad -(19)$$

for a Newtonian ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \qquad -(20)$$

For a precessing ellipse:

$$\left(\frac{v}{r\omega}\right)^2 - 1 = x^2\left(\frac{1+\epsilon}{1-\epsilon}\right)\frac{(r_{max}-r)(r-r_{min})}{r_{max}^2} \qquad -(21)$$

$$= x^2\left(\frac{1-\epsilon}{1+\epsilon}\right)\frac{(r_{max}-r)(r-r_{min})}{r_{min}^2}$$

and

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \qquad -(22)$$

For a circular orbit:

$$v = r\omega \qquad -(23)$$

$$r_{max} = r_{min} = r \qquad -(24)$$

$$\epsilon = 0 \qquad -(25)$$

5) Eq. (19) gives what appears to be a new equation for calculating $v/(r\omega)$ for a planet such as Earth, where:

$$\left(\frac{v}{r\omega}\right)^2 = 1 + \left(\frac{1+\epsilon}{1-\epsilon}\right)\frac{(r_{max}-r)(r-r_{min})}{r_{max}^2} \quad -(26)$$

in the Newtonian limit $(x = 1)$.

For Earth:

$$r_{max} = 1.52098232 \times 10^{12}\ m \quad -(27)$$

$$r_{min} = 1.47098290 \times 10^{12}\ m$$

$$\epsilon = 0.01671123$$

If we choose:

$$r = r_{av} = \frac{1}{2}(r_{max} + r_{min}) \quad -(28)$$

$$= 1.49598261 \times 10^{12}\ m$$

then

$$\boxed{\frac{v}{r_{av}\,\omega} = 1.00060} \quad -(29)$$

from eq. (26). For a circular orbit:

$$v = r\omega \quad -(30)$$

so

$$\frac{v}{r\omega} = 1, \quad -(31)$$

so the Earth's orbit is nearly circular as is well known.

6) In order to calculate the precession factor x for the Earth, eq. (21) must be used. For a given distance r between the Earth and the Sun, the linear orbital velocity v and the orbital angular velocity ω must be measured experimentally. The distances r_{max} and r_{min} are known accurately, so x can be found in this Minkowski theory. The orbital angular velocity ω is:

$$\omega = \frac{d\theta}{dt} \quad - (32)$$

and in one year the Earth rotates 2π radians, so ω can be found. An independent method must be found for measuring v experimentally. Then x can be found in this theory and compared with the experimental data.

Note carefully that general relativity is nowhere used in this theory.

However, for numerous other considerations general relativity is needed to construct a unified field theory. The theory of this note has many interesting philosophical consequences.