233(2): True Precessing Ellipse from the Minkowski Metric.

As shown in UFT149 and UFT150B the Minkowski metric produces the orbit:

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2}\right) \quad -(1)$$

where

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E}. \quad -(2)$$

The so called "Schwarzschild" metric produces the orbit:

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right) \quad -(3)$$

The true precessing ellipse is given by:

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad -(4)$$

where α is the half right latitude, ϵ the eccentricity and x the precession factor. From eq. (4)

$$\frac{dr}{d\theta} = \frac{\alpha\epsilon x\sin(x\theta)}{(1 + \epsilon\cos(x\theta))^2} \quad -(5)$$

$$= \frac{\epsilon x}{\alpha} r^2 \sin\theta$$

So

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon x}{\alpha}\right)^2 r^4 \sin^2(x\theta) \quad -(6)$$

From eq. (4):

2)

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(7)$$

Now use:

$$\cos^2(x\theta) + \sin^2(x\theta) = 1 \quad -(8)$$

So
$$\sin^2(x\theta) = 1 - \cos^2(x\theta) \quad -(9)$$

$$= 1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2 \quad -(10)$$

and
$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon x}{\alpha}\right)^2 r^4 \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right) \quad -(11)$$

From eqs. (1) and (11):

$$\boxed{x^2 = \frac{\left(\frac{\alpha}{\epsilon}\right)^2\left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2}\right)}{1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2}} \quad -(12)$$

For an ellipse:

$$\alpha = (1 - \epsilon^2)a_0 \quad -(13)$$
$$= (1 + \epsilon)r_{min}$$
$$= (1 - \epsilon)r_{max}$$

3) where a_0 is the semi ~~major~~ axis, r_{min} the distance of closest approach and r_{max} the distance of maximum separation.

Under these conditions, special relativity produces the true precessing ellipse.

As:

$$r \rightarrow \infty \quad -(14)$$

$$x^2 \rightarrow \frac{\alpha^2}{\epsilon^2 - 1}\left(\frac{1}{b^2} - \frac{1}{a^2}\right) \quad -(15)$$

$$= \frac{\alpha^2}{1-\epsilon^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

$$\boxed{x^2 \xrightarrow[r\rightarrow\infty]{} \left(\frac{(1-\epsilon)^2}{1-\epsilon^2}\right) r_{max}^2 \left(\frac{1}{a^2} - \frac{1}{b^2}\right)} \quad -(16)$$

For special relativity to reduce to the Newton theory then:

$$x \rightarrow 1 \quad -(17)$$

i.e.

$$\frac{1}{a^2} - \frac{1}{b^2} \xrightarrow[r\rightarrow\infty]{} \frac{1-\epsilon^2}{(1-\epsilon)^2}\frac{1}{r_{max}^2}$$

$$= \left(\frac{1-\epsilon}{1+\epsilon}\right)\frac{1}{r_{max}^2} \quad -(18)$$