

# 231(b): Potential Energy as a Result of Spacetime Dynamics

The Einstein energy equation can be written as:

$$\frac{E^2}{c^2} + \underline{p}^{(3)} \cdot \underline{p}_3 + \sqrt{2} (\underline{p}^{(1)} \cdot \underline{p}_1 - i \underline{p}^{(2)} \cdot \underline{p}_2) - (1)$$

$$= \frac{E^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = m^2 c^2$$

The fundamental tetrad definition is:

$$g_{\mu}^{\alpha} = \underline{e}^{\alpha} \cdot \underline{e}_{\mu} - (2)$$

where:

$$\underline{e}^{\alpha} = (\underline{e}^{(0)}, \underline{e}^{(1)}, \underline{e}^{(2)}, \underline{e}^{(3)}) - (3)$$

$$\underline{e}_{\mu} = (\underline{e}_0, \underline{e}_1, \underline{e}_2, \underline{e}_3) - (4)$$

A special case of eq. (2) is

$$g_{\mu}^{\mu} = \underline{e}^{\mu} \cdot \underline{e}_{\mu} - (5)$$

so  $g_0^0 = 1, g_i^i = -1, i = 1, 2, 3 - (6)$

Therefore:

$$\frac{E^2}{c^2} g_0^0 + p_x^2 g_1^1 + p_y^2 g_2^2 + p_z^2 g_3^3 - (7)$$

$$= \frac{E^2}{c^2} g_0^{(0)} + \sqrt{2} (p_x^2 g_1^{(1)} - i p_y^2 g_2^{(2)}) + p_z^2 g_3^{(3)}$$

$$= m^2 c^2$$

2) Considering the circular polar basis and Cartesian basis.

then:

$$\begin{bmatrix} \underline{e}^{(1)} \\ \underline{e}^{(2)} \end{bmatrix} = \begin{bmatrix} \sqrt{1}^{(1)} & \sqrt{2}^{(1)} \\ \sqrt{1}^{(2)} & \sqrt{2}^{(2)} \end{bmatrix} \begin{bmatrix} \underline{e}^1 \\ \underline{e}^2 \end{bmatrix} \quad - (8)$$

where

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} (\underline{i} - \underline{j}), \quad - (9)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}} (\underline{i} + \underline{j}), \quad - (10)$$

$$\underline{e}^1 = \underline{i}, \quad \underline{e}^2 = \underline{j}, \quad - (11)$$

so

$$\underline{e}^{(1)} = \sqrt{1}^{(1)} \underline{i} + \sqrt{2}^{(1)} \underline{j} \quad - (12)$$

$$\underline{e}^{(2)} = \sqrt{1}^{(2)} \underline{i} + \sqrt{2}^{(2)} \underline{j} \quad - (13)$$

Consider now the transformation:

$$\underline{e}^{(1)} \rightarrow e^{i\phi} \underline{e}^{(1)} \quad - (14)$$

where

$$\phi = \omega t - kZ. \quad - (15)$$

The effect of this transformation is to introduce rotation and translation of  $\underline{e}^{(1)}$ . The complex conjugate transform is:

$$\underline{e}^{(2)} \rightarrow e^{-i\phi} \underline{e}^{(2)} \quad - (16)$$

Eqs. (14) and (16) are equivalent to transforming the tetrad as follows:

3)

$$q_{-1}^{(1)} \rightarrow q_{-1}^{(1)} e^{i\phi}, \quad (17)$$

$$q_{+2}^{(1)} \rightarrow q_{+2}^{(1)} e^{i\phi}, \quad (18)$$

$$q_{-1}^{(2)} \rightarrow q_{-1}^{(2)} e^{-i\phi}, \quad (19)$$

$$q_{+2}^{(2)} \rightarrow q_{+2}^{(2)} e^{-i\phi}, \quad (20)$$

Eqs. (17) to (20) introduce the Lorentz transformation and curvature through the structure equations, so the spacetime is now that of general relativity.

The effect of eqs. (17) to (20) is:

$$p^2 \rightarrow p_x^2 e^{i\phi} + p_y^2 e^{-i\phi} + p_z^2 \quad (21)$$
$$:= \pi^2$$

It is now assumed that  $m^2 c^2$  is invariant under the transformations (17) to (20). So:

$$\frac{E^2}{c^2} - \pi^2 = \frac{E^2}{c^2} - p^2 = m^2 c^2 \quad (22)$$

$$\boxed{E^2 - c^2 \pi^2 = E^2 - c^2 p^2 = m^2 c^4} \quad (23)$$

Furthermore, it is assumed that:

4)

$$F = E - V, \quad \underline{\pi} = \underline{p} - \underline{p}_1 \quad (24)$$

where  $V$  is a potential energy of spacetime, and  $\underline{p}_1$  is a linear momentum of spacetime. The quantities  $\underline{V}$  and  $\underline{p}_1$  are introduced by making the frame of reference a dynamic quantity as in eqs. (17) to (20).

Therefore we obtain the relativistic and generally covariant Hamilton-Jacobi equation:

$$\pi^\mu \pi_\mu = m^2 c^2 \quad (25)$$

where

$$\pi^\mu = \left( \frac{E}{c}, \underline{\pi} \right) \quad (26)$$

From eqs. (23) and (24):

$$(E - V)^2 = c^2 (\underline{p} - \underline{p}_1) \cdot (\underline{p} - \underline{p}_1) + m^2 c^4 \quad (27)$$

This can now be reduced to a Schrödinger equation and used to study the effect of spacetime energy-momentum on UENP.