229(9): Approximate Solution for $r > R$.

In the region: $r > R$ — (1)

then
$$V = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)} + \frac{Z_1 Z_2 e^2}{r} \quad -(2)$$

If $a \gg r - R$ — (3)

then
$$V \sim -\frac{V_0}{2+\frac{r-R}{a}} + \frac{C}{r} \quad -(4)$$

where
$$C = Z_1 Z_2 e^2 \quad -(5)$$

The integral is:
$$\int_a^b \left(-\frac{V_0}{2+\frac{r}{a}-\frac{R}{a}} + \frac{C}{r} - E \right)^{1/2} dr \quad -(6)$$

and this has an analytical solution:
$$\int \left(-\frac{V_0}{2+\frac{r-R}{a}} + \frac{C}{r} - E \right)^{1/2} dr$$

2)

$$= \int \left(- \frac{V_0}{b + \frac{r}{a}} + \frac{C}{r} - E \right)^{1/2} dr \quad -(7)$$

where $$b = 2 - \frac{R}{a} \quad -(8)$$

i.e. $$\underline{I} = \int \left(- \frac{A}{B + r} + \frac{C}{r} - E \right)^{1/2} dr \quad -(9)$$

where
$$\left. \begin{aligned} A &= a V_0, \\ B &= ab = a\left(2 - \frac{R}{a}\right) = 2a - R \\ C &= Z_1 Z_2 e^2 \end{aligned} \right\} -(10)$$

It may be that eq. (9) has an analytical solution. Wolfram integrator runs out of time trying to integrate it, but Maxima or Mathematica might be able to do so.