

LOW ENERGY NUCLEAR REACTION: QUANTUM TUNNELLING AND  
SPACETIME ABSORPTION.

by

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ABSTRACT

The conditions under which low energy nuclear reaction occurs can be optimized by a straightforward application of the Schroedinger equation with a realistic model of the internuclear potential. Starting from the ECE wave equation, the effect of spacetime absorption can be considered. The conditions for low energy reaction are defined as total energy  $E$  of the incoming atom much less than the potential energy  $V$  of interaction. Quantum tunnelling is optimized when the transmission coefficient  $T$  is maximized. For a Coulomb barrier it is demonstrated that  $T$  is maximized for  $E \ll V$  when the mass of the incoming atom is maximized. A more realistic potential is considered, made up of a combination of Coulomb repulsion force between nuclear protons of two different atoms, and a strong nuclear attraction force.

Keywords: ECE wave equation, optimal condition for low energy nuclear reaction.

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## 1 INTRODUCTION

In recent papers of this series of 229 papers to date {1 - 10} the theory of low energy nuclear reaction (LENR) has been considered in detail. In this paper the optimal conditions for LENR are defined using a straightforward procedure based on the Schroedinger equation with a realistic model potential. The optimal conditions for LENR are defined by maximum  $T$  for  $E \ll V$ , where  $T$  is the transmission coefficient of quantum tunnelling,  $E$  is the total energy and  $V$  the potential energy of the Schroedinger equation, the non relativistic quantum limit of the ECE wave equation {1 - 10}. In addition, the effect of wave absorption is considered on the LENR process. In some working devices {11} a phonon wave is applied to the reaction. In ECE theory this is a wave of spacetime within a proportionality factor, and ECE theory also considers the absorption of momentum. It has been shown in previous papers of this series (notably UFT 158 ff. on [www.aias.us](http://www.aias.us)) that conventional Compton scattering, absorption and Raman scattering theory collapses without correct consideration of momentum transfer.

In Section 2 a realistic model potential is defined which consists of Coulombic repulsion {12} between the protons of the nuclei of two interacting atoms 1 and 2, and strong nuclear attraction {13, 14} with the Woods Saxon mean field model. As atom 1 approaches atom 2 it first meets the Coulomb barrier. It is shown in Section 3, using computer algebra, that it can quantum tunnel effectively through this barrier when  $E \ll V$ . The coefficient  $T$  is maximized when the mass of the incoming atom is maximized. The Coulomb repulsion inside the fused entity defined in Section 2 is modelled as in conventional theory of nuclear fusion as the Coulombic repulsion inside a sphere. In the fused entity there is also a strong nuclear attraction between nucleons, both protons and neutrons. This is modelled with the well known Woods Saxon mean field potential. The complete potential is the sum of the

Coulombic repulsion between protons and the strong attraction between protons and neutrons. As atom 1 approaches atom 2, the complete potential goes through a positive maximum before decreasing to a negative minimum in the fused entity made up of atoms 1 and 2 combined. The entity decomposes almost immediately to give the products of the fusion reaction.

In order for low energy nuclear reaction to occur at low total energy  $E$ , the fused entity must be formed by quantum tunnelling of atom 1 into atom 2. The well known WKB approximation {12} used in the previous paper UFT228 of this series ([www.aias.us](http://www.aias.us)) is extended to the complete potential and the transmission coefficient of quantum tunnelling evaluated numerically in Section 3. The effect of absorption is developed in Section 2 from the ECE wave equation {1 - 10}, which is a well known and generally covariant generalization to unified field theory of the Schroedinger equation.

## 2. ABSORPTION AND QUANTUM TUNNELLING IN LENR.

Consider the ECE wave equation:

$$(\square + R) \gamma_{\mu}^a = 0 \quad - (1)$$

which can be expanded as:

$$(\square + R) \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} = 0 \quad - (2)$$

Here  $\gamma_{\mu}^a$  is the Cartan tetrad, and  $R$  is defined {1 - 10} by geometry. Eq. (2) can be reduced to the Klein Gordon equation:

$$\left( \square + \left( \frac{mc}{\hbar} \right)^2 \right) \psi = 0 \quad - (3)$$

where  $m$  is the particle mass,  $c$  the speed of light and  $\hbar$  the reduced Planck constant. Eq.

( 3 ) can be deduced from the Einstein energy equation:

$$E^2 = p^2 c^2 + m^2 c^4 \quad - (4)$$

using the Schroedinger postulates:

$$E = i\hbar \frac{d}{dt}, \quad \underline{p} = -i\hbar \underline{\nabla}, \quad - (5)$$

so it follows that:

$$\left( \frac{mc}{\hbar} \right)^2 = \left( \frac{\omega}{c} \right)^2 - \kappa^2 \quad - (6)$$

and

$$\left( \square + \frac{\omega^2}{c^2} - \kappa^2 \right) \psi = 0. \quad - (7)$$

Eq. ( 7 ) can also be written as:

$$(E^2 - c^2 p^2) \psi = m^2 c^4 \psi = \hbar^2 (\omega^2 - \kappa^2 c^2) \psi \quad - (8)$$

which is an example of wave particle dualism. It can be linearized using:

$$E^2 - m^2 c^4 = (E - mc^2)(E + mc^2) \quad - (9)$$

so:

$$(E - mc^2) \psi = \left( \frac{c^2 p^2}{E + mc^2} \right) \psi. \quad - (10)$$

In the non relativistic approximation:

$$E \rightarrow mc^2 \quad - (11)$$

so:

$$(E - mc^2) \psi = \frac{p^2}{2m} \psi \quad - (12)$$

Eq. ( 12 ) is the free particle Schrodinger equation, usually written as:

$$\frac{p^2}{2m} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi = E \psi \quad - (13)$$

i.e. the total energy, written E in the Schrodinger equation, is the total relativistic energy of the free particle minus its rest energy:

$$E = T = (\gamma - 1) mc^2 \quad - (14)$$

where T is the relativistic kinetic energy of the free particle. The free particle Schrodinger equation may be written therefore as:

$$\nabla^2 \psi = -\frac{2mE}{\hbar^2} \psi \quad - (15)$$

where the mass is:

$$m = \frac{\hbar}{c^2} (\omega^2 - \kappa^2 c^2)^{1/2} \quad - (16)$$

If the de Broglie Einstein postulates are assumed then:

$$E = \hbar \omega = \gamma mc^2 \quad - (17)$$

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v} \quad - (18)$$

where:

$$\gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (19)$$

is the Lorentz factor. Here v is the velocity of the free particle,  $\omega$  is its angular frequency and  $\kappa$  its wavenumber.

The process of absorption of spacetime energy and momentum can be described as a change of mass:

$$m \rightarrow m + m_1 = \frac{\hbar}{c^2} \left[ (\omega^2 - \kappa^2 c^2)^{1/2} + (\omega_1^2 - \kappa_1^2 c^2)^{1/2} \right] \quad (20)$$

so the effective mass of the particle increases. The same conclusion holds for the reduced mass of two interacting masses,  $m_1$  and  $m_2$ :

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad (21)$$

In the presence of a potential energy  $V$  the free particle Schroedinger equation becomes:

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = E \psi \quad (22)$$

i.e.:

$$\nabla^2 \psi = -\frac{2m}{\hbar^2} (E - V) \psi \quad (23)$$

In the well known {12} WKB approximation the transmission coefficient for quantum tunnelling from Eq. (23) is {12}:

$$T = \frac{4}{\left( 2\theta + \frac{1}{2\theta} \right)^2} \quad (24)$$

where:

$$\theta = \exp \left( \frac{(2\mu)}{\hbar} \int_a^b (\nabla(r) - E)^{1/2} dr \right) \quad (25)$$

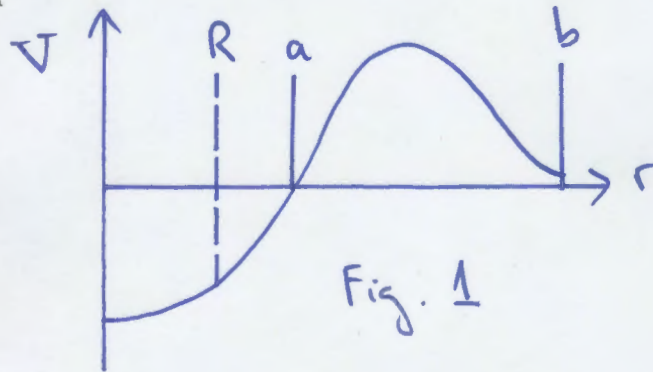
where the integral is evaluated between two points on the potential. In general, this quantum tunnelling process may be accompanied by quantum absorption described as follows:

$$\mu \rightarrow \mu + \frac{\hbar}{c^2} (\omega_1^2 - \kappa_1^2 c^2)^{1/2} \quad - (26)$$

In conventional nuclear fusion theory the potential in one dimension is:

$$V = - \frac{V_0}{1 + \exp\left(\frac{r-R}{a_N}\right)} + V_C \quad - (27)$$

which is the sum of Coulombic repulsion and strong nuclear attraction. This potential is sketched in Fig. 1



The Coulombic repulsion in conventional fusion theory is:

$$V_C = Z_1 Z_2 e^2 / r, \quad r > R, \quad - (28)$$

$$V_C = \frac{Z_1 Z_2 e^2}{R} \left( 3 - \left( \frac{r}{R} \right)^2 \right), \quad r < R, \quad - (29)$$

where there are  $Z_1$  protons in atom 1 and  $Z_2$  protons in atom 2. The region  $r$  less than  $R$  defines the interior of the fused entity of radius  $R$ . Therefore the interior is modelled as a charged sphere of radius  $R$ . Outside the fused entity in the region defined by  $r$  greater than  $R$  the usual Coulomb law is used conventionally. On the ECE level {1 - 10} the Coulomb law is changed.

The potential due to the strong nuclear force between protons and neutrons is

modelled conventionally in the mean field approximation by the Woods Saxon potential:

$$V = - \frac{V_0}{1 + \exp\left(\frac{r-R}{a_N}\right)} \quad - (30)$$

The minus sign means that the strong force is a force of attraction between both protons and neutrons, i.e. between all nucleons, either in the separate nuclei 1 or 2, or in the fused nucleus. Here  $V_0$  is the well depth, and  $a$  is the surface thickness of the nucleus.

Therefore the transmission coefficient is worked out in general using the combined potential ( 27 ). In the region where the repulsive part dominates the potential reduces to:

$$V \rightarrow Z_1 Z_2 e^2 / r \quad - (31)$$

and as shown in Section 3 the transmission coefficient in this case can be expressed as:

$$T = \frac{16y}{16y^2 + 8y + 1} \quad - (32)$$

where:

$$y = \exp\left(\frac{(\frac{2mE}{\hbar^2})^{1/2} \pi a}{\hbar}\right) \quad - (33)$$

Maximum transmission occurs at exactly:

$$y = \frac{1}{4}, \quad - (34)$$

and at this value:

$$mE = \frac{(\log_e 4)^2 \hbar^2}{2\pi^2 a^2} \quad - (35)$$

Since  $a$  varies very slowly with  $Z$ , then:



$$E \propto \frac{1}{m} \quad (36)$$

for optimal transmission, the heavier the element the less the required energy.

### 3. RESULTS AND DISCUSSION

Section by Dr. Horst Eckardt and Dr. Douglas Lindstrom

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