228(12): Simple Relativistic Theory of Quantum Tunnelling

From eqs. (24) of note 228(7) define:

$$\kappa_1^2 = k^2 + \kappa_0^2 = \frac{\gamma m}{\hbar^2} E \quad -(1)$$

$$\kappa_1^2 = \kappa^2 + \kappa_0^2 = \frac{\gamma m}{\hbar^2}(E - V_0) \quad -(2)$$

where

$$\kappa_0^2 = \left(\frac{mc}{\hbar}\right)^2, \quad -(3)$$

and

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(4)$$

where v is the particle velocity. It can be shown as follows that eq. (1) is the Einstein energy equation:

$$E^2 = p^2c^2 + m^2c^4. \quad -(5)$$

Write eq. (1) as:

$$\gamma m E = \hbar^2 k^2 + \hbar^2 \kappa_0^2 \quad -(6)$$

$$= p^2 + p_0^2$$

and use

$$E = \gamma m c^2. \quad -(7)$$

Therefore:

$$\gamma m c^2 E = E^2 = c^2p^2 + c^2p_0^2 \quad -(8)$$

i.e.

$$E^2 = c^2p^2 + m^2c^4 \quad -(9)$$

Q.E.D.

2) The relativistic transmission coefficient is:

$$T = \frac{8\kappa_1^2 k_1^2}{(k_1^2+\kappa_1^2)\cosh(4\kappa_1 a) - (\kappa_1^4 + k_1^4 - 6\kappa_1^2 k_1^2)} \quad -(10)$$

where

$$k_1^2 = \frac{\gamma m}{\hbar^2} E \quad -(11)$$

$$\kappa_1^2 = \frac{\gamma m}{\hbar^2}(E - V_0) \quad -(12)$$

Using:

$$E = \gamma m c^2, \quad -(13)$$

then

$$k_1^2 = \left(\frac{\gamma m c}{\hbar}\right)^2 \quad -(14)$$

$$\kappa_1^2 = \left(\frac{\gamma m c}{\hbar}\right)^2 - \frac{\gamma m V_0}{\hbar^2} \quad -(15)$$

So for a given m, a and V_0, T can be plotted against v, the velocity of the incoming particle.
