226(8): General Structure of Field Interaction Theory

On the classical level:

$$p^\mu \rightarrow p^\mu - p_1^\mu - p_2^\mu - p_3^\mu - \dots \quad -(1)$$

and at the semi classical level:

$$i\hbar \partial^\mu \rightarrow i\hbar \partial^\mu - p_1^\mu - p_2^\mu - p_3^\mu - \dots \quad -(2)$$

For example, to develop a theory of the interaction of electromagnetism and gravitation:

$$p_1^\mu = e A^\mu \quad -(3)$$

$$p_2^\mu = m \Phi^\mu \quad -(4)$$

To develop a theory of the interaction of the electromagnetic and weak field:

$$p_1^\mu = e A^\mu \quad -(5)$$

$$p_2^\mu = g W^\mu \quad -(6)$$

and for particle interaction:

$$p_1^\mu = \left(\frac{E_1}{c}, \underline{p_1} \right). \quad -(7)$$

In general:

$$(E - E_1 - E_2 - \dots)^2 = c^2 (p - p_1 - p_2 - \dots)^2 + m^2 c^4, \quad -(8)$$

which is the Einstein energy equation with eq. (1).

Therefore:

2)

$$E - E_1 - E_2 - \ldots = \frac{c^2(p - p_1 - p_2 - \ldots)^2}{E - E_1 - E_2 - \ldots} + \frac{m^2c^4}{E - E_1 - E_2 - \ldots} \quad -(9)$$

Add mc^2 to left side:

$$E - E_1 - E_2 - \ldots + mc^2 = \frac{c^2(p - p_1 - p_2 - \ldots)^2}{E - E_1 - E_2 - \ldots} + \frac{m^2c^4}{E - E_1 - E_2 - \ldots} + mc^2 \quad -(10)$$

For small perturbations this is approximated by:

$$E - E_1 - E_2 - \ldots + mc^2 \sim \frac{c^2}{E}(p - p_1 - p_2 - p_3 - \ldots)^2 + \frac{m^2c^4}{E} + mc^2 \quad -(11)$$

In the non-relativistic limit:

$$E = \gamma mc^2 \rightarrow mc^2 \quad -(12)$$

so:

$$2mc^2 - E_1 - E_2 - \ldots = \frac{c^2}{E}(p - p_1 - p_2 - \ldots)^2 + 2mc^2 \quad -(13)$$

i.e.

$$(2mc^2 - E_1 - E_2 - \ldots)E = c^2(p - p_1 - p_2 - \ldots)^2 + 2mc^2E \quad -(14)$$

and:

$$E = c^2(\underline{p} - \underline{p}_1 - \underline{p}_2 - \underline{p}_3 \ldots)(2mc^2 - E_1 - E_2 - \ldots)^{-1}(\underline{p} - \underline{p}_1 - \underline{p}_2 - \underline{p}_3 \ldots) + \left(\frac{2mc^2}{2mc^2 - E_1 - E_2 - \ldots}\right)E \quad -(15)$$

3) For small perturbation:

$$E - mc^2 = \frac{1}{2m}\left(\underline{p} - \underline{p}_1 - \underline{p}_2 \ldots\right)\left(1 - \frac{E_1 - E_2 - \ldots}{2mc^2}\right)^{-1}\left(\underline{p} - \underline{p}_1 - \underline{p}_2 \ldots\right) \quad -(16)$$

i.e.

$$T = \frac{1}{2m}\left(\underline{p} - \underline{p}_1 - \underline{p}_2 - \ldots\right)\left(1 + \frac{E_1 + E_2 + E_3 + \ldots}{2mc^2}\right)\left(\underline{p} - \underline{p}_1 - \underline{p}_2 - \ldots\right) \quad -(17)$$

$$= E - mc^2$$

This quantizes to the Schrodinger type equation:

$$\boxed{\frac{1}{2m}\left(\hat{\underline{p}} - \underline{p}_1 - \underline{p}_2 - \ldots\right)\left(1 + \frac{E_1 + E_2 + E_3 + \ldots}{2mc^2}\right)\left(\hat{\underline{p}} - \underline{p}_1 - \underline{p}_2 - \ldots\right)\phi = T\phi} \quad -(18)$$

where

$$\hat{\underline{p}} = -i\hbar\,\underline{\nabla} \quad -(19)$$

This is the same type of series of approximations that lead to the g factor of the electron, the anomalous Zeeman effect, the Landé factor, ESR, NMR, MRI, the Thomas factor and Darwin term. It can be seen that gravitation will have an effect on all these phenomena, and so will

4) the weak field and strong field. There are other problems that can be considered with eq. (18), to give many new insights.

Considering the first term on the right hand side of eq. (18):

$$T\psi = \left(\frac{1}{2m}\underline{\sigma}\cdot(\underline{\hat{p}}-\underline{p}_1-\underline{p}_2\ldots)\,\underline{\sigma}\cdot(\underline{\hat{p}}-\underline{p}_1-\underline{p}_2\ldots)\right)\psi \quad -(20)$$

As in note 225(4):

$$\underline{\sigma}\cdot(\underline{\hat{p}}-\underline{p}_1-\underline{p}_2\ldots)\,\underline{\sigma}\cdot(\underline{\hat{p}}-\underline{p}_1-\underline{p}_2\ldots) \quad -(21)$$

$$= \underline{\sigma}\cdot\underline{\hat{p}}\,\underline{\sigma}\cdot\underline{\hat{p}} - \underline{\sigma}\cdot\underline{\hat{p}}\,\underline{\sigma}\cdot\underline{p}_1 - \underline{\sigma}\cdot\underline{\hat{p}}\,\underline{\sigma}\cdot\underline{p}_2$$

$$- \underline{\sigma}\cdot\underline{p}_1\,\underline{\sigma}\cdot\underline{\hat{p}} + \underline{\sigma}\cdot\underline{p}_1\,\underline{\sigma}\cdot\underline{p}_1 + \underline{\sigma}\cdot\underline{p}_1\,\underline{\sigma}\cdot\underline{p}_2$$

$$- \underline{\sigma}\cdot\underline{p}_2\,\underline{\sigma}\cdot\underline{\hat{p}} + \underline{\sigma}\cdot\underline{p}_2\,\underline{\sigma}\cdot\underline{p}_1 + \underline{\sigma}\cdot\underline{p}_2\,\underline{\sigma}\cdot\underline{p}_2$$

Here:

$$\underline{\sigma}\cdot\underline{\hat{p}}\,\underline{\sigma}\cdot\underline{p}_1 = \underline{\hat{p}}\cdot\underline{p}_1 + i\underline{\sigma}\cdot\underline{\hat{p}}\times\underline{p}_1$$

$$\underline{\sigma}\cdot\underline{p}_1\,\underline{\sigma}\cdot\underline{\hat{p}} = \underline{p}_1\cdot\underline{\hat{p}} + i\underline{\sigma}\cdot\underline{p}_1\times\underline{\hat{p}}$$

$$\underline{\sigma}\cdot\underline{\hat{p}}\,\underline{\sigma}\cdot\underline{p}_2 = \underline{\hat{p}}\cdot\underline{p}_2 + i\underline{\sigma}\cdot\underline{\hat{p}}\times\underline{p}_2$$

$$\underline{\sigma}\cdot\underline{p}_2\,\underline{\sigma}\cdot\underline{\hat{p}} = \underline{p}_2\cdot\underline{\hat{p}} + i\underline{\sigma}\cdot\underline{p}_2\times\underline{\hat{p}}$$

$$\underline{\sigma}\cdot\underline{p}_1\,\underline{\sigma}\cdot\underline{p}_2 = \underline{p}_1\cdot\underline{p}_2 + i\underline{\sigma}\cdot\underline{p}_1\times\underline{p}_2$$

$$\underline{\sigma}\cdot\underline{p}_2\,\underline{\sigma}\cdot\underline{p}_1 = \underline{p}_2\cdot\underline{p}_1 + i\underline{\sigma}\cdot\underline{p}_2\times\underline{p}_1$$

5) In this Dirac type approximation there are cross terms between fields at the semi classical level. These can be worked out systematically to give a variety of spectral and other effects. The basic structure is that of the fermion equation:

$$\left((E - E_1 - E_2 - \ldots) + c\underline{\sigma} \cdot (\underline{p} - \underline{p}_1 - \underline{p}_2 \ldots)\right)\phi^L = mc^2 \phi^R$$
$$\left((E - E_1 - E_2 - \ldots) - c\underline{\sigma} \cdot (\underline{p} - \underline{p}_1 - \underline{p}_2 \ldots)\right)\phi^R = mc^2 \phi^L \qquad -(22)$$

In particle interaction theory the same overall structure conserved.