226(7): Reduction to the Schroedinger Equation.

In the last note the ECE fermion equation was reduced to:

$$(\hat{E} - mc^2)\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi + \underline{\sigma}\cdot(\underline{p} - e\underline{A})\frac{e\phi}{4m^2c^2}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi \quad -(1)$$

The presence of c^2 in the denominator of the second term on the RHS means that, to an excellent approximation:

$$(\hat{E} - mc^2)\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi \quad -(2)$$

$$= \frac{1}{2m}\left(p^2 + e^2A^2 - e(\underline{A}\cdot\underline{p} + \underline{p}\cdot\underline{A}) - ie(\underline{\sigma}\cdot\underline{A}\times\underline{p} + \underline{\sigma}\cdot\underline{p}\times\underline{A})\right)\phi.$$

The Schroedinger equation is obtained by writing this equation as:

$$(E - mc^2)\phi = \frac{1}{2m}\underline{\sigma}\cdot(\hat{\underline{p}} - e\underline{A})\underline{\sigma}\cdot(\hat{\underline{p}} - e\underline{A})\phi \quad -(3)$$

i.e. E is regarded as a function and:

$$\hat{p} = -i\hbar\hat{\underline{\nabla}}. \quad -(4)$$

In the absence of interaction:

$$\underline{A} = \underline{0} \quad -(5)$$

so:

$$\frac{1}{2m}\hat{p}^2\phi = (E - mc^2)\phi. \quad -(6)$$

Here the relativistic kinetic energy is:

$$T = E - mc^2 = (\gamma - 1)mc^2 \quad -(7)$$

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So:
$$\hat{H}\phi = T\phi \quad -(8)$$
where the Hamiltonian operator is:
$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 \quad -(9)$$
In the non-relativistic limit:
$$T \rightarrow \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad -(10)$$
so
$$\hat{H}\phi = \frac{p^2}{2m}\phi \quad -(11)$$
In order to work out the effect of interaction on the kinetic energy, eq. (3) is developed using eq. (4). Therefore eq. (8) becomes:
$$-\frac{e\hbar}{2m}\left(\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}+\underline{\nabla}\times\underline{A}\right)\right)\phi - \frac{\hbar^2}{2m}\nabla^2\phi = T\phi \quad -(12)$$
i.e.
$$-\frac{\hbar^2}{2m}\nabla^2\phi = \left(T+\frac{e\hbar}{2m}\left(\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}+\underline{\nabla}\times\underline{A}\right)\right)\right)\phi \quad -(13)$$
Therefore the kinetic energy is augmented by an interaction term. In the case of eq. (13) it is an interaction with the electromagnetic field, but it can be an interaction with any type of external field. <u>In ECE theory</u> all <u>fields</u> come <u>from</u> spacetime itself.

Eq. (13) can be developed as follows:

$$-\frac{\hbar^2}{2m}\nabla^2\phi = \left(T + \frac{e\hbar}{2m}\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}\phi + \underline{\nabla}\times(\underline{A}\phi)\right)\right)$$

$$= \left(T + \frac{e\hbar}{2m}\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}\phi + (\underline{\nabla}\times\underline{A})\phi + (\underline{\nabla}\phi)\times\underline{A}\right)\right)$$

$$= \left(T + \frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{\nabla}\times\underline{A}\right)\phi \quad -(14)$$

This equation can be written as:

$$\hat{H}\phi = E\phi \quad -(15)$$

where

$$E = T + V \quad -(16)$$

and

$$V = \frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{\nabla}\times\underline{A} \quad -(17)$$

On the u(1) level:

$$\underline{B} = \underline{\nabla}\times\underline{A} \quad -(18)$$

is the magnetic flux density. On the ECE level eq. (18) is augmented by a spin connection term. So:

$$V = \frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{B} \quad -(19)$$

giving the Landé factor, anomalous Zeeman effect, half integral spin of the electron, ESR, NMR and MRI.

Interaction with other Fields

The above theory is described as semi-classical because it is based on:

$$p^\mu \rightarrow p^\mu + eA^\mu \quad -(20)$$

and

$$p^\mu \rightarrow i\hbar \partial^\mu \quad -(21)$$

so:

$$\boxed{i\hbar\partial^\mu \rightarrow i\hbar\partial^\mu + eA^\mu} \quad -(22)$$

In general:

$$i\hbar\partial^\mu \rightarrow i\hbar\partial^\mu + F^\mu \quad -(23)$$

where F^μ is any field. On the ECE level eq. (23) is true for all states of polarization in:

$$F^a_\mu = F^{(0)} q^a_\mu \quad -(24)$$

where q^a_μ is the Cartan tetrad. For the electromagnetic field:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(25)$$

For the gravitational field:

$$\Phi^a_\mu = \Phi^{(0)} q^a_\mu \quad -(26)$$

For the weak nuclear field:

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$$W_\mu^a = W^{(0)} q_\mu^a \quad -(27)$$

For the strong nuclear field:

$$S_\mu^a = S^{(0)} q_\mu^a \quad -(28)$$

These are all examples of the general spacetime field of eq. (24), i.e. are all example of a unified physics.

For example, if an electron interacts with an electromagnetic field and a gravitational field:

So

$$i\hbar \partial^\mu \rightarrow i\hbar \partial^\mu + eA^\mu + m\Phi^\mu \quad -(29)$$

where

$$\Phi^\mu = \left(\frac{\Phi}{c}, \Phi \right) \quad -(29)$$

and where m is the mass of the electron.

If an electron interacts with the matter field F^μ of another particle, then:

$$i\hbar \partial^\mu \rightarrow i\hbar \partial^\mu + g F^\mu \quad -(30)$$

where g is the coupling constant. So:

$$p^\mu = g F^\mu \quad -(31)$$

where p^μ is the four momentum of the matter field.

The interaction is therefore:

$$i\hbar \partial^\mu \rightarrow i\hbar \partial^\mu + p^\mu \quad -(32)$$

where

$$p^\mu = \left(\frac{E}{c}, \underline{p} \right) \quad -(33)$$

The description of this process in the Schroedinger equation is therefore:

$$\hat{H}\phi = E\phi \quad -(34)$$

where:

$$E = T + V \quad -(35)$$

and:

$$\boxed{V = \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \times \underline{p}} \quad -(36)$$

So the kinetic energy of the electron becomes:

$$T \rightarrow T + \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \times \underline{p} \quad -(37)$$

where $\underline{p}$ is the momentum of the matter field. In ECE theory:

$$p_\mu^a = p^{(0)} q_\mu^a \quad -(38)$$

So for every index a the four momentum of the matter field is a spacetime property. A nuclear reaction may occur in this way.