226(5): General Particle Interaction with the Fermion Equation.

Consider the interaction and transmutation of two particles of masses m_1 and m_2 to produce two particles of mass m_3 and m_4. In a relativistic classical treatment total energy is conserved:

$$E_1 + E_2 = E_3 + E_4 \quad -(1)$$

where

$$E_n = \gamma_n m_n c^2 \quad -(2)$$

$$n = 1, \ldots, 4$$

and where γ_n are the Lorentz factors:

$$\gamma_n = \left(1 - \frac{v_n^2}{c^2}\right)^{-1/2}. \quad -(3)$$

Total momentum is also conserved:

$$\underline{p}_1 + \underline{p}_2 = \underline{p}_3 + \underline{p}_4 \quad -(4)$$

where

$$\underline{p}_n = \gamma_n m_n \underline{v}_n, \quad -(5)$$

$$n = 1, \ldots, 4$$

The ECE fermion equation for particles 1 and 2 on the relativistic quantum level are:

$$\left(\hat{E}_1 + c\,\underline{\sigma} \cdot \hat{\underline{p}}_1\right)\phi_1^L = m_1 c^2 \phi_1^R \quad -(6)$$

$$\left(\hat{E}_1 - c\,\underline{\sigma} \cdot \hat{\underline{p}}_1\right)\phi_1^R = m_1 c^2 \phi_1^L \quad -(7)$$

2) and:

$$(\hat{E}_2 + c\underline{\sigma}\cdot\hat{\underline{p}}_2)\phi_2^L = m_2 c^2 \phi_2^R \quad -(8)$$

$$(\hat{E}_2 - c\underline{\sigma}\cdot\hat{p}_2)\phi_2^R = m_2 c^2 \phi_2^L \quad -(9)$$

with similar equations for m_3 and m_4.

The rigorously quantum development of the particle interaction is obtained by adding eqs. (1) and (3) to give:

$$\hat{E}_1\phi_1^L + \hat{E}_2\phi_2^L + (c\underline{\sigma}\cdot\hat{\underline{p}}_1)\phi_1^L + (c\hat{\underline{\sigma}}\cdot\hat{p}_2)\phi_2^L$$
$$= m_1 c^2 \phi_1^R + m_2 c^2 \phi_2^R \quad -(10)$$

and similarly for eqs. (2) and (4).

The semi-classical development relies on the minimal prescription:

$$\hat{p}_1 \rightarrow \hat{p}_1 + p_2 \quad -(11)$$

where $\hat{p}_1$ is an operator and p_2 is classical. $-(12)$

So:

$$((\hat{E}_1 + E_2) + c\underline{\sigma}\cdot(\hat{\underline{p}}_1 + p_2))\phi_1^L = m_1 c^2 \phi_1^R$$

$$((\hat{E}_1 + E_2) - c\underline{\sigma}\cdot(\hat{p}_1 + p_2))\phi_1^R = m_1 c^2 \phi_1^L \quad -(13)$$

and $$((\hat{E}_2 + E_1) + c\underline{\sigma}\cdot(\hat{p}_2 + p_1))\phi_2^L = m_2 c^2 \phi_2^R \quad -(14)$$

3) $$\left(\left(\hat{E}_2 + E_1\right) - c\underline{\sigma}\cdot\left(\hat{p}_2 + p_1\right)\right)\phi_2^R = m_2 c^2 \phi_2^L \quad - (15)$$

So there are two simultaneous equations:

$$\left(\hat{E}_1 + E_2\right)^2 \phi_1^R = \left(c^2 \underline{\sigma}\cdot\left(\underline{\hat{p}_1} + \underline{p_2}\right)\underline{\sigma}\cdot\left(\hat{p}_1 + p_2\right) + m_1^2 c^4\right)\phi_1^R \quad - (16)$$

and
$$\left(\hat{E}_2 + E_1\right)^2 \phi_2^R = \left(c^2 \underline{\sigma}\cdot\left(\hat{p}_2 + p_1\right)\underline{\sigma}\cdot\left(\hat{p}_2 + p_1\right) + m_2^2 c^4\right)\phi_2^R \quad - (17)$$

However, if E_2 is classical, only eq. (16) need be considered.

After the reaction and transmutation:

$$\left(\hat{E}_3 + E_4\right)^2 \phi_3^R = \left(c^2 \underline{\sigma}\cdot\left(\underline{\hat{p}_3} + \underline{p_4}\right)\underline{\sigma}\cdot\left(\underline{\hat{p}_3} + p_4\right) + m_3^2 c^4\right)\phi_3^R \quad - (18)$$

This is a general theory that can be applied to chemical and nuclear reactions.