226(3): Fermion Equation for Interacting Mass and Classical Limits.

In previous notes it was shown that the wave equation for the interacting mass m is:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad -(1)$$

As in previous work this factorizes into the ECE fermion equation:

$$\pi_\mu \phi \sigma^\mu = mc \sigma^1 \phi \quad -(2)$$

which can be written as the two simultaneous equations:

$$(E + c\underline{\sigma}\cdot\underline{p})\phi^L = mc^2 \phi^R \quad -(3)$$

$$(E - c\underline{\sigma}\cdot\underline{p})\phi^R = mc^2 \phi^L \quad -(4)$$

where

$$\phi^R = \begin{bmatrix} \phi_1^R \\ \phi_2^R \end{bmatrix}, \quad \phi^L = \begin{bmatrix} \phi_1^L \\ \phi_2^L \end{bmatrix} \quad -(5)$$

Eqs. (3) and (4) can be written as:

$$(E^2 - c^2 \underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p})\phi^R = m^2c^4 \phi^R \quad -(6)$$

$$(E^2 - c^2 \underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p})\phi^L = m^2c^4 \phi^L, \quad -(7)$$

the classical limit of which is:

$$E^2 = c^2p^2 + m^2c^4 \quad -(8)$$

which is the Einstein energy equation with

2) positive energy. The fermion equation (2) removes the problem of negative energy from the Dirac equation.

Eq. (6) can be written as:

$$c^2 \underline{\sigma} \cdot \underline{p}\ \underline{\sigma} \cdot \underline{p}\ \phi^R = \left(E^2 - m^2c^4\right)\phi^R \quad -(9)$$

i.e.

$$c^2 p^2 \phi^R = \left(E^2 - m^2c^4\right)\phi^R, \quad -(10)$$

$$p^2 \phi^R = \left(\frac{E^2}{c^2} - m^2c^2\right)\phi^R, \quad -(11)$$

where

$$E = \gamma mc^2. \quad -(12)$$

So:

$$p^2 \phi^R = \left(\gamma^2 - 1\right)m^2c^2\phi^R$$

$$= m^2c^2\left(\left(1 - \frac{v^2}{c^2}\right)^{-1} - 1\right)\phi^R \quad -(13)$$

In the non relativistic limit:

$$v << c \quad -(14)$$

so

$$p^2 \phi^R \doteq m^2 v^2 \phi^R \quad -(15)$$

i.e.

$$\boxed{p = mv.} \quad -(16)$$

Eq. (6) can also be written as:

3)
$$\left(E - \frac{c^2}{E}\,\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p}\right)\phi^R = \frac{m^2c^4}{E}\,\phi^R \quad -(17)$$
$$-(18)$$

Add mc^2 to left side:
$$\left(E + mc^2 - \frac{c^2}{E}\,\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p}\right)\phi^R = \left(\frac{m^2c^4}{E} + mc^2\right)\phi^R$$

In the limit (14):
$$E \longrightarrow mc^2, \quad -(19)$$

so
$$\left(2mc^2 - \frac{c^2}{E}\,\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p}\right)\phi^R = 2mc^2\phi^R \quad -(20)$$

i.e.
$$\left(2mc^2E - c^2\,\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p}\right)\phi^R = 2mc^2E\,\phi^R$$
$$= 2m^2c^4\,\phi^R \quad -(21)$$

so
$$E\phi^R = \left(mc^2 + \frac{1}{2m}\,\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p}\right)\phi^R \quad -(22)$$

or
$$\left(E - mc^2\right)\phi^R = \frac{1}{2m}\,p^2\,\phi^R. \quad -(23)$$

This means that:
$$T\phi^R = (\gamma - 1)\,mc^2\,\phi^R = \frac{p^2}{2m}\,\phi^R \quad -(24)$$

where T is the relativistic kinetic energy.

4) In the classical limit:

$$T = (\gamma - 1) mc^2 = \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) mc^2$$

$$\sim \left(1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) mc^2$$

$$= \frac{1}{2} mv^2 \quad -(25)$$

So eq. (24) reduces to:

$$T\phi^R = \frac{1}{2} mv^2 \phi^R = \frac{p^2}{2m} \phi^R \quad -(26)$$

i.e.

$$\boxed{T = \frac{1}{2} mv^2 = \frac{p^2}{2m}} \quad -(27)$$

This checks that the basic structure of the relativistic quantum theory is correct. These equations give all the details of the low energy nuclear reaction with the exception of radiative corrections. The interacting mass always replaces the measured mass, introducing extra energy due to spacetime.