GENERAL ECE THEORY OF FIELD AND PARTICLE INTERACTION : APPLICATION TO LOW ENERGY NUCLEAR REACTION (LENR)

by

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ABSTRACT

The general ECE theory is developed of field interaction and particle interaction on the classical and quantum relativistic level using the minimal prescription. The theory conserves total energy / momentum and charge / current density, and is based on the development of the tetrad postulate of Cartan geometry into the EEC wave equation and fermion equation. The latter is developed for any kind of interaction between fields or between particles or particles and fields. In ECE theory all of these interactions are phenomena of spacetime represented by geometry. The general theory is applied to reproducible and repeatable experimental data from low energy nuclear reactions.

Keywords: ECE theory, general interaction between fields and particles, low energy nuclear reaction.

UFT 226

1. INTRODUCTION

In papers of this series {1 - 10} it has been shown that the received opinion on particle interaction becomes wildly erroneous when conservation of energy and momentum are correctly considered. These are papers UFT158 ff. of www.aias.us, also published in ref. (1). The fundamental theory of particle interaction in the received opinion has collapsed. In order to remedy this disaster for standard physics a new approach was suggested in UFT181 and UFT182 based on the ECE wave equation {1 - 10}. The latter was derived in the early papers of this series from the tetrad postulate of Cartan geometry {11}. In UFT172 to UFT174 on www.aias.us the fermion equation was derived from the ECE wave equation. The fermion equation is equivalent to the chiral representation of the Dirac equation but dispenses with the need for Dirac matrices. It uses the two by two tetrad matrix. The fermion equation does not lead to unphysical negative energy, so has this great advantage over the Dirac equation. In Section 2 the fermion equation is developed into a general ECE theory of field field, particle field, and particle particle interaction using a generalized minimal prescription. This general theory can be applied to a wide range of problems. It conserves total energy / momentum, and total charge / current density. It is a unified field theory and it is generally covariant, and can be used with all four fundamental fields: gravitation, electromagnetism, weak and strong nuclear. It can also be applied to particle particle interaction or matter field /matter field interaction, or particle / matter field interaction, for example scattering, chemical reactions, annihilation and transmutation, fission and fusion. In Section 3 it is applied to specific examples of low energy nuclear reaction (LENR). The experimental data in LENR are generally accepted to be reproducible and repeatable, and LENR devices giving a new source of energy are expected to be available in the near future. So it is important to understand LENR with ECE theory, the first generally accepted and generally covariant unified field theory.

2. GENERAL ECE THEORY

This section should be read as usual in conjunction with the background notes posted along with this paper on www.aias.us. The background notes provide comprehensive scholarly detail of which this paper is a synopsis.

Consider two particles of four momenta p^μ and p_1^μ :

$$p^\mu = \left(\frac{E}{c}, \underline{p} \right), \; p_1^\mu = \left(\frac{E_1}{c}, \underline{p}_1 \right) \quad -(1)$$

In the semi classical development:

$$p^\mu = i\hbar \partial^\mu \quad -(2)$$

where:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla} \right). \quad -(3)$$

In the minimal prescription the interaction is described by:

$$p^\mu \rightarrow p^\mu + p_1^\mu \quad -(4)$$

So:

$$E \rightarrow E + E_1 \quad -(5)$$

$$\underline{p} \rightarrow \underline{p} + \underline{p}_1 \quad -(5a)$$

where E is the total relativistic energy:

$$E = \gamma m c^2 \quad -(6)$$

and where $\underline{p}$ is the relativistic momentum:

$$\underline{p} = \gamma m \underline{v} . \quad -(7)$$

The Lorentz factor is defined by:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(8)$$

where $\underline{v}$ is the velocity of a particle of mass m and where c is the speed of light in vacuo. Eq. (7) implies {12} the Einstein energy equation:

$$E^2 = p^2 c^2 + m^2 c^4 \quad -(9)$$

which can be written as:

$$E^2 - m^2 c^4 = (E - mc^2)(E + mc^2) = c^2 p^2 . \quad -(10)$$

The relativistic kinetic energy {12} is defined as:

$$T = E - mc^2 = (\gamma - 1) mc^2 = \frac{c^2 p^2}{E + mc^2} . \quad -(11)$$

So the relativistic kinetic energy is:

$$T = \left(\frac{\gamma^2}{\gamma - 1}\right) mv^2 \quad -(12)$$

and reduces in the non-relativistic limit:

$$\gamma \rightarrow 1 \quad -(13)$$

to the classical non relativistic kinetic energy of the particle:

$$T = \frac{1}{2} mv^2 . \quad -(14)$$

From Eqs. (4) and (9):

$$(E+E_1)^2 = c^2(p+p_1)^2 + m^2c^4 \quad -(15)$$

This is the classical relativistic description of particle interaction with the minimal prescription. From Eq. (15):

$$(E+E_1)^2 - m^2c^4 = c^2(p+p_1)^2 \quad -(16)$$

so:

$$T = E + E_1 - mc^2 = \frac{c^2(p+p_1)^2}{E + E_1 + mc^2} \quad -(17)$$

is the relativistic kinetic energy of a particle of mass m interacting with a particle of mass m

It can be expressed as:

$$T = m\,\frac{(\gamma v + \gamma_1 v_1)^2}{1 + \gamma + \gamma_1} \quad -(18)$$

where:

$$\gamma_1 = \left(1 - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(19)$$

where v_1 is the velocity of particle m_1 .

This classical relativistic theory is a limit of the ECE fermion equation, which is derived from Cartan geometry. The concepts of particle mass m and m_1 are limits of the more general R factor of the ECE wave equation as described in UFT181 and UFT182 and preceding papers. In general, ECE theory allows mass to vary. The analysis of UFT158 ff. Shows that the concept of fixed particle mass in the received opinion is completely untenable. It is well known that the Dirac equation can be used to describe phenomena such as the g factor of the electron, the Landé factor, the anomalous Zeeman effect, electron spin resonance

(ESR), nuclear magnetic resonance (NMR), magnetic resonance imaging (MRI), the Thomas factor, spin orbit coupling and the Darwin effect. However the approximations used to claim these results are very carefully selected. This selection of approximation is illustrated next on the classical relativistic level. The fermion equation produces all these phenomena given the same selection of approximation. With contemporary computers such approximations are not needed and a much more thorough analysis can be initiated.

The approximations start by writing eq. (15) as:

$$E + E_1 = \frac{c^2 (p + p_1)^2}{E + E_1} + \frac{m^2 c^4}{E + E_1} \, . \quad -(20)$$

Add mc^2 to both sides:

$$E + E_1 + mc^2 = \frac{c^2 (p + p_1)^2}{E + E_1} + \frac{m^2 c^4}{E + E_1} + mc^2 \, . . \quad -(21)$$

Assume that:

$$E_1 << E \, . \quad -(22)$$

In the denominators on the right hand side of Eq. (21) assume that

$$E + E_1 \sim E \quad -(23)$$

to obtain:

$$E + E_1 + mc^2 = \frac{c^2 (p + p_1)^2}{E} + \frac{m^2 c^4}{E} + mc^2 \, . \quad -(24)$$

Next assume that in the classical non relativistic limit:

$$E = \gamma mc^2 \longrightarrow mc^2 \, . \quad -(25)$$

Use this approximation in eq. (24) in the following selected manner:

$$2mc^2 + E_1 = \frac{c^2}{E}(p+p_1)^2 + \frac{m^2c^4}{mc^2} + mc^2 = \frac{c^2}{E}(p+p_1)^2 + 2mc^2 \quad - (26)$$

When quantized these are the approximations used by Dirac and his contemporaries. They are not very satisfactory because they are selected approximations, i.e. are not used consistently through the equations. A factor of two has appeared and this is the basis of the claim that the Dirac equation gives the g factor and Thomas factor. In reality, the factor two has been very carefully selected from the theory to give the "right" result.

Next, Eq. (26) is rearranged as:

$$E = \frac{c^2(p+p_1)^2}{2mc^2+E_1} + \frac{2mc^2 E}{2mc^2+E_1} \quad - (27)$$

In the second term on the right hand side of this equation it is assumed that:

$$E_1 << 2mc^2, \quad E \sim mc^2 \quad - (28)$$

to obtain:

$$E = \frac{c^2(p+p_1)^2}{2mc^2+E_1} + mc^2 \quad - (29)$$

Therefore the relativistic kinetic energy of the interacting particles is

$$T = E - mc^2 = \frac{1}{2m}(p+p_1)^2\left(1+\frac{E_1}{2mc^2}\right)^{-1} \quad - (30)$$

Finally assume that:

$$\left(1+\frac{E_1}{2mc^2}\right)^{-1} \sim 1 - \frac{E_1}{2mc^2} \quad - (31)$$

to obtain:

$$T = \frac{1}{2m}\left(p+p_1\right)^2\left(1-\frac{E_1}{2mc^2}\right). \quad -(32)$$

Comparing Eqs. (32) and (17) it is seen that Eq. (17) has been approximated by use of Eq. (26), so Eq. (17) becomes:

$$T = E + E_1 - mc^2 \sim \frac{c^2\left(p+p_1\right)^2}{2mc^2+E_1}. \quad -(33)$$

This equation is further approximated by:

$$T = E + E_1 - mc^2 \sim E - mc^2 \quad -(34)$$

to give Eq. (32).

In order to quantize this theory the fermion equation {1 - 10} is used:

$$\left(\left(E+E_1\right) + c\,\underline{\sigma}\cdot\left(\underline{p}+\underline{p_1}\right)\right)\phi^L = mc^2\phi^R \quad -(35)$$

$$\left(\left(E+E_1\right) - c\,\underline{\sigma}\cdot\left(\underline{p}+\underline{p_1}\right)\right)\phi^R = mc^2\phi^L \quad -(36)$$

where the right and left spinors are defined by:

$$\phi^L = \begin{bmatrix}\phi_1^L\\ \phi_2^L\end{bmatrix},\quad \phi^R = \begin{bmatrix}\phi_1^R\\ \phi_2^R\end{bmatrix}. \quad -(37)$$

It follows that:

$$\left(\left(E+E_1\right)^2 - c^2\,\underline{\sigma}\cdot\left(\underline{p}+\underline{p_1}\right)\underline{\sigma}\cdot\left(\underline{p}+\underline{p_1}\right)\right)\phi^L = m^2c^4\phi^L \quad -(38)$$

and similarly for ϕ^R. The carefully selected approximations described already on the classical level are implemented as follows, giving a range of phenomena in this general theory of interaction.

Write Eq. (38) as:

$$(E+E_1)\phi^L = \left(\underline{\sigma} \cdot (\underline{p}+\underline{p_1}) \left(\frac{c^2}{E+E_1}\right) \underline{\sigma} \cdot (\underline{p}+\underline{p_1}) + \frac{m^2c^4}{E+E_1} \right) \phi^L \quad -(39)$$

Add mc^2 to each side:

$$(E+E_1+mc^2)\phi^L = \left(\underline{\sigma} \cdot (\underline{p}+\underline{p_1}) \left(\frac{c^2}{E+E_1}\right) \underline{\sigma} \cdot (\underline{p}+\underline{p_1}) + \frac{m^2c^4}{E+E_1} + mc^2 \right) \phi^L \quad -(40)$$

Approximate in the same way as described already on the classical level to find that

$$\hat{H}\phi^L = T\phi^L \quad -(41)$$

where:

$$T = E - mc^2 \quad -(42)$$

and

$$\hat{H} = \frac{1}{2m} \underline{\sigma} \cdot (\underline{p}+\underline{p_1}) \left(1 - \frac{E_1}{2mc^2}\right) \underline{\sigma} \cdot (\underline{p}+\underline{p_1}) \quad -(43)$$

is the hamiltonian operator. In the momentum representation of quantum mechanics:

$$\underline{p} = -i\hbar \underline{\nabla} \quad -(44)$$

where $\hbar$ is the reduced Planck constant. The hamiltonian operator is therefore:

$$\hat{H} = \hat{H}_1 + \hat{H}_2 \quad -(45)$$

where:

$$\hat{H}_1 = \frac{1}{2m}\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}+\underline{p}_1\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}+\underline{p}_1\right) \quad -(46)$$

and

$$\hat{H}_2 = -\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}+\underline{p}_1\right)\frac{E_1}{4m^2c^2}\left(-i\hbar\underline{\nabla}+\underline{p}_1\right). \quad -(47)$$

Consider for the sake of illustration the interaction of the U(1) electromagnetic potential A^μ with an electron. Then the $\hat{H}_1$ operator is claimed in the received opinion to give the g factor of the electron, the anomalous Zeeman effect, ESR, NMR and MRI. As we have argued, this claim is based on very carefully selected approximation designed to introduce the critical factor two. The second hamiltonian $\hat{H}_2$ gives the Thomas factor, spin orbit coupling and the Darwin term.

All these phenomena will have their equivalents in the general ECE theory being developed here. In addition there is no need to adhere to the approximation procedures of an earlier era because of available computational methods. So a multitude of new phenomena emerge from the theory, even on this semi classical level.

In Eq. (43):

$$\underline{\sigma}\cdot\left(\underline{p}+\underline{p}_1\right)\underline{\sigma}\cdot\left(\underline{p}+\underline{p}_1\right) = p^2 + p_1^2 + \underline{p}_1\cdot\underline{p} + \underline{p}\cdot\underline{p}_1 + i\underline{\sigma}\cdot\left(\underline{p}_1\times\underline{p}+\underline{p}\times\underline{p}_1\right) \quad -(48)$$

so the first type of hamiltonian becomes:

$$\hat{H}_1 = -\frac{\hbar^2}{2m}\nabla^2 + \frac{p_1^2}{2m} + \frac{i\hbar}{2m}\left(\underline{p}_1\cdot\underline{\nabla}+\underline{\nabla}\cdot\underline{p}_1\right) + \frac{\hbar}{2m}\underline{\sigma}\cdot\left(\underline{p}_1\times\underline{\nabla}+\underline{\nabla}\times\underline{p}_1\right) \quad -(49)$$

and operates as follows:

$$\hat{H}_1 \phi^L = T \phi^L \qquad -(50)$$

to give energy eigenvalues. Note carefully that:

$$(\underline{p}_1 \cdot \underline{\nabla}) \phi^L = \underline{p}_1 \cdot \underline{\nabla} \phi^L , \qquad -(51)$$

$$\underline{\nabla} \cdot \underline{p}_1 \phi^L = \underline{\nabla} \cdot (\underline{p}_1 \phi^L) = (\underline{\nabla} \cdot \underline{p}_1) \phi^L + \underline{p}_1 \cdot \underline{\nabla} \phi^L \qquad -(52)$$

using the Leibnitz Theorem. Similarly:

$$(\underline{p}_1 \times \underline{\nabla}) \phi^L = \underline{p}_1 \times (\underline{\nabla} \phi^L) \qquad -(53)$$

and

$$\underline{\nabla} \times \underline{p}_1 \phi^L = (\underline{\nabla} \times \underline{p}_1) \phi^L + (\underline{\nabla} \phi^L) \times \underline{p}_1 . \qquad -(54)$$

Using:

$$\underline{p}_1 \times (\underline{\nabla} \phi^L) + (\underline{\nabla} \phi^L) \times \underline{p}_1 = \underline{0} \qquad -(55)$$

the hamiltonian operator becomes:

$$\hat{H}_1 = -\frac{\hbar^2}{2m} \nabla^2 + \frac{p_1^2}{2m} + \frac{i\hbar}{2m} \left(\underline{\nabla} \cdot \underline{p}_1 + 2 \underline{p}_1 \cdot \underline{\nabla} \right) + \frac{\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \times \underline{p}_1 . \qquad -(56)$$

This result may be applied to a large number of phenomena within the approximation procedure used. For example, the minimal prescription for the interaction of an electron with a classical U(1) electromagnetic field is:

$$p^\mu \rightarrow p^\mu + e A^\mu . \qquad -(57)$$

On the ECE level the minimal prescription is:

$$p_\mu^a \rightarrow p_\mu^a + eA_\mu^a \quad -(58)$$

and the ECE level leads to a large number of new insights {1 - 10}, bringing in to consideration the spin connection. It has been shown in UFT131 ff of www.aias.us that the U(1) description collapses completely when antisymmetry is correctly applied, so is used here for illustration only. Eq. (58) means that for each state of polarization a, the minimal prescription applies. On the U(1) level the hamiltonian operator (56) becomes:

$$\hat{H}_1 = -\frac{\hbar^2}{2m}\nabla^2 + \frac{e^2A^2}{2m} + \frac{ie\hbar}{2m}\left(\underline{\nabla}\cdot\underline{A} + 2\underline{A}\cdot\underline{\nabla}\right) + \frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{\nabla}\times\underline{A} \quad -(59).$$

and this operator generates interaction energy eigenvalues. It can be used to describe Aharanov Bohm effects and to describe the interaction of the background potential of ECE theory with an electron.

In order to describe the absorption of a photon on the U(1) level the following equation is used:

$$eA^\mu = \hbar\kappa^\mu. \quad -(60)$$

Here:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right), \quad \kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa}\right) \quad -(61)$$

where ϕ is the scalar potential, $\underline{A}$ is the vector potential, ω the angular frequency and

$\underline{\kappa}$ the wave vector. In UFT162 of www.aias.us it was shown that the conventional theory of absorption collapses due to neglect of conservation of momentum, but in this theory total momentum is conserved.

In the generally covariant form of this theory, the concept of mass is replaced by the curvature R using the Hamilton Jacobi equation:

$$(p^\mu - \hbar \kappa^\mu)(p_\mu - \hbar \kappa_\mu) = m_0^2 c^2 \quad -(62)$$

as in UFT182 of www.aias.us. , where Eq. (62) was written as:

$$p^\mu p_\mu = \hbar^2 R_1 + m_0^2 c^2. \quad -(63)$$

Consider the four momentum p_1^μ of particle 1 interacting with matter wave 2 defined by the wave four vector κ_2^μ . Particle 1 is also a matter wave by the Planck / de Broglie postulate:

$$p_1^\mu = \hbar \kappa_1^\mu. \quad -(64)$$

In UFT182 it was shown that the interaction is described by:

$$\left(\square + R_2 + \left(\frac{m_{10} c}{\hbar} \right)^2 \right) \phi_1 = 0 \quad -(65)$$

where the R_2 parameter is:

$$R_2 = \left(\frac{m_2 c}{\hbar} \right)^2 \quad -(66)$$

and is defined by the concept of interacting mass:

$$m_2 = \frac{\hbar}{c} \left[2 \left(\frac{\omega_1 \omega_2}{c^2} - \kappa_1 \kappa_2 \right) - \left(\frac{\omega_2^2}{c^2} - \kappa_2^2 \right) \right]^{1/2}. \quad -(67)$$

This concept was introduced to account for the findings of UFT158 ff., which show that the concept of fixed particle mass is untenable completely. In Eq. (65) therefore m_{10} denotes the measured mass. Eq. ( ) can be written as:

$$\left(\Box + \left(\frac{\underline{M}_2 c}{\hbar}\right)^2\right)\phi_1 = 0 \quad -(68)$$

where

$$\underline{M}_2 = \left(m_2^2 + m_{10}^2\right)^{1/2} \quad -(69)$$

and is an example of the ECE wave equation:

$$(\Box + R)\, q^a_\mu = 0 \quad -(70)$$

which is factorized in UFT172 to UFT174 to the fermion equation. This method is further developed in the accompanying note 226(2).

Therefore in this general ECE theory it is possible to think of a quantum of spacetime energy being absorbed during a reaction. This idea generalizes the Planck concept of a quantum of electromagnetic energy, the photon. A low energy nuclear reaction (LENR) can be exemplified as follows:

$${}^{64}Ni + p = {}^{63}Cu + 2n. \quad -(71)$$

Here ${}^{64}Ni$ has 36 neutrons and 28 protons, and ${}^{63}Cu$ has 34 neutrons and 29 protons. So ${}^{64}Ni$ is transmuted into ${}^{63}Cu$ with the release of two neutrons. The theory must explain why this reaction occurs at low energies. The classical description results in a scattering process:

$${}^{64}Ni(i) + p(i) = {}^{64}Ni(f) + p(f) \quad -(72)$$

and no transmutation. The proton p would be repelled by the ${}^{64}Ni$ nucleus, and no neutrons would be released. However, in LENR, nickel is observed to be transmuted to copper with the release of usable energy. Total energy must be conserved, so there must be a source of energy that is not accounted for in received physics. In the theory of UFT181 on www.aias.us :

$$p^\mu \rightarrow p^\mu - \hbar \kappa^\mu \quad -(73)$$

and the reaction (71) is described by the Hamilton Jacobi equation:

$$(p^\mu - \hbar \kappa^\mu)(p_\mu - \hbar \kappa_\mu) = m_0^2 c^2 \quad -(74)$$

where m_0 is the measured mass of the free nickel atom. Using the method of UFT181, Eq. (74) may be written as:

$$\left(\square + R_1 + \left(\frac{m_0 c}{\hbar}\right)^2\right)\phi = 0 \quad -(75)$$

where:

$$R_1 = \left(\frac{mc}{\hbar}\right)^2 \quad -(76)$$

and where m is the interacting mass:

$$m = \frac{\hbar}{c}\left(\frac{\omega^2}{c^2} - \kappa^2\right)^{1/2} \quad -(76)$$

This is a property of spacetime, and ω and $\underline{\kappa}$ are the angular frequency and wave number of the proton matter wave, a property of spacetime. The total mass of the nickel atom during interaction therefore increases to:

$$\underline{M} = (m^2 + m_0^2)^{1/2} \quad -(77)$$

and this critical mass has concomitant energy:

$$E_0 = \underline{M} c^2 \quad -(78)$$

so that a nuclear reaction occurs. The process may be thought of as an absorption of a quantum of spacetime by the nickel nucleus, so that dissociation occurs with the release of neutrons. In Section 3 further examples of LENR are discussed.

3. LOW ENERGY NUCLEAR REACTIONS

Section by Dr. Horst Eckardt and Dr. Douglas Lindstrom

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REFERENCES

{1} M. W. Evans, Ed., J. Found. Phys. Chem., (Cambridge International Science Publishing, www.cisp-publishing.com, , CISP, 2011 onwards, six editions a year).

{2} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CISP, Spring 2011).

{3} M .W Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis Academic, 2005 to 2011) in seven volumes.

{4} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis, 2007, Spanish translation by Alex Hill on www.aias.us).

{5} M .W. Evans, H. Eckardt and D. W. Lindstrom, plenary and papers in the Serbian

Academy of Sciences, from 2010 onwards.

{6} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley, 1992, 1993, 1997, 2001) in two editions and six volumes.

{7} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

{8} M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer 1994 to 2002) in ten volumes, hardback and softback.

{9} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).

{10} K. Pendergast, "The Life of Myron Evans" (CISP, 2011).

{11} S. P. Carroll, "Spacetime and Geometry: an Intorduction to Geneal Relativity" (Addison Wesley, New York, 2004), chapter three.

{12} J. B. Marion and S. T. Thornton, "Classical Dynamics of Particles and Systems" (Harcourt Brace, New York, 1988, 3rd edition).