225(4): Effect of the Weak Field on Atomic and Molecular Spectra.

Consider the fermion equation in the format:

$$\hat{E}_3 \phi^R = H_3 \phi^R \quad -(1)$$

$$\hat{E}_3 \phi^L = H_3 \phi^L \quad -(2)$$

In order to consider the weak nuclear interaction the left hand spinor is extended to:

$$\phi^L \rightarrow \begin{bmatrix} \phi^L \\ \nu \end{bmatrix} \quad -(3)$$

but the right hand spinor is unaffected. So eq. (2) is:

$$\hat{E}_3 \begin{bmatrix} \phi^L \\ \nu \end{bmatrix} = H_3 \begin{bmatrix} \phi_L \\ \nu \end{bmatrix} \quad -(4)$$

For the electron part of this equation:

$$\hat{E}_3 \phi^L = \frac{1}{2m} \underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi} \quad -(5)$$

where

$$\underline{\pi} = \underline{p} - e\underline{A} - g\underline{W} \quad -(6)$$

in the notation of previous notes. Therefore m is the mass of the electron

1) For real $\underline{\Pi}$:

$$\underline{\sigma}\cdot\underline{\Pi}\,\underline{\sigma}\cdot\underline{\Pi} = \underline{\Pi}\cdot\underline{\Pi} + i\underline{\sigma}\cdot\underline{\Pi}\times\underline{\Pi} \quad -(7)$$

So

$$\underline{\sigma}\cdot\underline{\Pi}\,\underline{\sigma}\cdot\underline{\Pi} = \underline{\sigma}\cdot(\underline{p}-e\underline{A}-g\underline{W})\,\underline{\sigma}\cdot(\underline{p}-e\underline{A}-g\underline{W})$$

$$= \underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p} - e\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{A} - g\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{W}$$

$$- e\underline{\sigma}\cdot\underline{A}\,\underline{\sigma}\cdot\underline{p} + e^2\underline{\sigma}\cdot\underline{A}\,\underline{\sigma}\cdot\underline{A} + eg\underline{\sigma}\cdot\underline{A}\,\underline{\sigma}\cdot\underline{W}$$

$$- g\underline{\sigma}\cdot\underline{W}\,\underline{\sigma}\cdot\underline{p} + ge\underline{\sigma}\cdot\underline{W}\,\underline{\sigma}\cdot\underline{A} + g^2\underline{\sigma}\cdot\underline{W}\,\underline{\sigma}\cdot\underline{W} \quad -(8)$$

Here:

$$\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{A} = \underline{p}\cdot\underline{A} + i\underline{\sigma}\cdot\underline{p}\times\underline{A} \quad -(9)$$

$$\underline{\sigma}\cdot\underline{A}\,\underline{\sigma}\cdot\underline{p} = \underline{A}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{A}\times\underline{p} \quad -(10)$$

$$\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{W} = \underline{p}\cdot\underline{W} + i\underline{\sigma}\cdot\underline{p}\times\underline{W} \quad -(11)$$

$$\underline{\sigma}\cdot\underline{W}\,\underline{\sigma}\cdot\underline{p} = \underline{W}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{W}\times\underline{p} \quad -(12)$$

$$\underline{\sigma}\cdot\underline{A}\,\underline{\sigma}\cdot\underline{W} = \underline{A}\cdot\underline{W} + i\underline{\sigma}\cdot\underline{A}\times\underline{W} \quad -(13)$$

$$\underline{\sigma}\cdot\underline{W}\,\underline{\sigma}\cdot\underline{A} = \underline{W}\cdot\underline{A} + i\underline{\sigma}\cdot\underline{W}\times\underline{A} \quad -(14)$$

As in note 225(2) there are terms such as:

$$\hat{H}_W \phi^L = -\frac{ig}{2m}\left(\underline{\sigma}\cdot\underline{W}\times\hat{\underline{p}} + \underline{\sigma}\cdot\hat{\underline{p}}\times\underline{W}\right)\phi^L$$

$$= \frac{\hbar g}{2m}\left(\underline{\sigma}\cdot\left(\underline{W}\times\underline{\nabla} + \underline{\nabla}\times\underline{W}\right)\right)\phi^L$$

$$= -\frac{g\hbar}{2m}\left(\underline{\sigma}\cdot\underline{\nabla}\times\underline{W}\right)\phi^L \quad -(7)$$

So the energy for the left handed electron is:

$$\boxed{E_L = -\frac{\hbar}{2m}\left(e\,\underline{\sigma}\cdot\underline{B} + g\,\underline{\sigma}\cdot\underline{\nabla}\times\underline{W}\right)} \quad -(8)$$

For the right handed electron:

$$E_R = -\frac{\hbar e}{2m}\,\underline{\sigma}\cdot\underline{B} \quad -(9)$$

This means that the following effects are slightly different for right and left handed electrons: Faraday effect, Zeeman effect, ESR, NMR, FMR, Spin-orbit coupling, Thomas precession, Darwin term, and similar.
