1) 175(8): Particle on a Ring Anti Commutator

In note 175(7) it was shown that the anti-commutator plays a fundamental role in quantum mechanics. So for the particle on a ring the computational research is to work out:

1) $$\{x, \frac{d}{dx}\}\phi = x\frac{d\phi}{dx} + \frac{d}{dx}(x\phi) = 2\frac{d\phi}{dx} + \phi$$

where

$$\phi = \left(\frac{1}{2\pi}\right)^{1/2}\left(\exp(im_J\phi) + \exp(-im_J\phi)\right)$$

where:

$$\phi = \cos^{-1}\frac{x}{(x^2+y^2)^{1/2}} = \sin^{-1}\frac{y}{(x^2+y^2)^{1/2}}.$$

Similarly:

2) $$\{x, \frac{d}{dy}\}\phi = x\frac{d\phi}{dy} + \frac{d}{dy}(x\phi) = x\frac{d\phi}{dy} + \left(\frac{dx}{dy}\right)\phi + x\frac{d\phi}{dy}$$

$$= 2x\frac{d\phi}{dy} + \left(\frac{dx}{dy}\right)\phi$$

3) $$\{y, \frac{d}{dx}\}\phi = 2y\frac{d\phi}{dx} + \left(\frac{dy}{dx}\right)\phi$$

4) $$\{y, \frac{d}{dy}\}\phi = 2\frac{d\phi}{dy} + \phi.$$